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TRACKING ACCURACY PREDICTION PROGRAM

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CAPABILITY GUIDE

TRACKING ACCURACY PREDICTION PROGRAM (TAPP MOD I)

(Revised)

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INTRODUCTION

In the design of space flight trajectories and hardware it is necessary to determine the answers to questions of the following nature:

In a nominal operation, how accurately will the spacecraft trajectory or certain terminal variables be determined from tracking, as a function of time throughout the flight?

What is the sensitivity of this nominal accuracy to the number and location of tracking stations, the quantities measured, observation noise models, data biases, etc.?

What is the effect of uncertainties in physical constants and station locations on the nominal accuracy?

What is the effect of a midcourse correction on the orbit determination accuracy?

The Tracking Accuracy Prediction Program (TAPP) has been designed specifically for the statistical analysis of such pre-flight orbit determination problems (as distinct from the operational processing of data to determine a particular orbit once a flight has occurred). In developing the program, emphasis has been placed on computational speed, capability of handling a wide range of problems, and ease of future program modification. To these ends, the following features have been included: For speed trajectory computation is based on a three-dimensional, multi-center, patched conic model so that no integration is required.* In addition the ephemerides of celestial bodies are computed from formulas rather than by table look-up, and frequent tracking observations are interpolated from a basic mesh of time steps.

* Extensive comparison at STL of the results of such models and the results of "exact" integrating programs has shown good agreement for both lunar and interplanetary flights.

The orbit computational scheme is completely general in that it can deal with all types of conics with essentially no alterations in the formulas. No difficulties are encountered in such troublesome cases as parabolic, near parabolic, circular, and zero inclination orbits. This flexibility is made possible by the use of the Cartesian coordinates at a fixed epoch as the orbital elements along with Herrick's unified parameters (Reference 2) for finding the spacecraft position velocity vector on the orbit.

A variety of observation types may be simulated, including range, range rate, hour angle, declination, elevation, and azimuth from earth based stations; planetary diameters and star-planet sightings from the spacecraft; and range and range rate from a lunar-based station. Rise and set times are computed, allowing the user to specify the observations to be taken by convenient "rules" and placing the burden of generating the observation times on the program. A number of noise models and station locations are pre-stored in the program and may be specified by a code number. Other models and station locations may of course be entered as input quantities. The effects of uncertainties in station locations, physical constants, and biases may be studied. Up to 25 orbital elements and non-orbital parameters may be solved for, and the effect of executing a midcourse maneuver may be simulated. A choice of five printout formats is provided covering trajectory variables, midcourse quantities, and tracking matrices, and varying in the amount and type of detail printed out.

The program described in this report (TAPP Mod I) was designed for the tracking analysis of flights containing a single midcourse correction. An extended version (TAPP Mod II) is under development which will allow simulatio of n midcourse and terminal guidance corrections. This latter program employs a Monte Carlo method of analysis and is intended for combined trackin guidance "mission analysis", including studies of midcourse fuel requirements relative efficiencies of guidance logics, and the study of adaptive correctior systems.

This report is a revision of an earlier document, "Computer Program Guide, Tracking Accuracy Prediction Program (TAPP MOD I)", by L. Wong, A. S. Liu, M. C. Fujisaki, and O. Senda. It has been modified to reflect changes in the program and to eliminate errors.

The first part of the report presents a functional block diagram of the program and includes a general description of each major program block. The second part is a series of appendices which describe the input requirements, output options, and print formats, and give all equations used in the various program blocks.

GENERAL PROGRAM DESCRIPTION

In the computation of orbits, it is assumed that an orbit is determined as a function of time from the equations of motion if the combined initial position and velocity vector, x_o, is given at one instant, t_o. In practice x_o is never known exactly but can be estimated from observations made along the orbit. Such observations are subject to random noise which introduces fluctuations into the calculated values of x_o.

The object of the present program is the evaluation of orbit determination accuracy on the basis of a given noise model and the details of observations along the orbit. For our purpose, the accuracy criterion is the covariance matrix of a set of variables which are known functions of x_o. Usually these variables are taken to be the impact parameter vector with respect to a target planet or in the case of elliptic motion, the spacecraft position vector at a fixed time.

In order to find the covariance matrix referred to above, the method of least squares is used to estimate the initial position and velocity vector, x_o, from the observations. The covariance matrix for x_o is obtained from the weighted least squares matrices. The covariance matrix for functions of x_o can then be obtained by a linear transformation. [That is, except for effects of physical constants which will be discussed later].

Briefly, the tasks required for finding the covariance matrix of impact errors are outlined in block diagram form in Figure 1.

In addition to the primary purpose of tracking accuracy evaluation, the program may sometimes be used to compute from x_o,

a) the approximate spacecraft trajectory and a set of auxiliary quantities given in Appendix 10,

b) the spacecraft rise and set times from a number of stations over a time span of interest,

c) target sensitivity coefficients for midcourse maneuvers at prescribed points in the orbit,

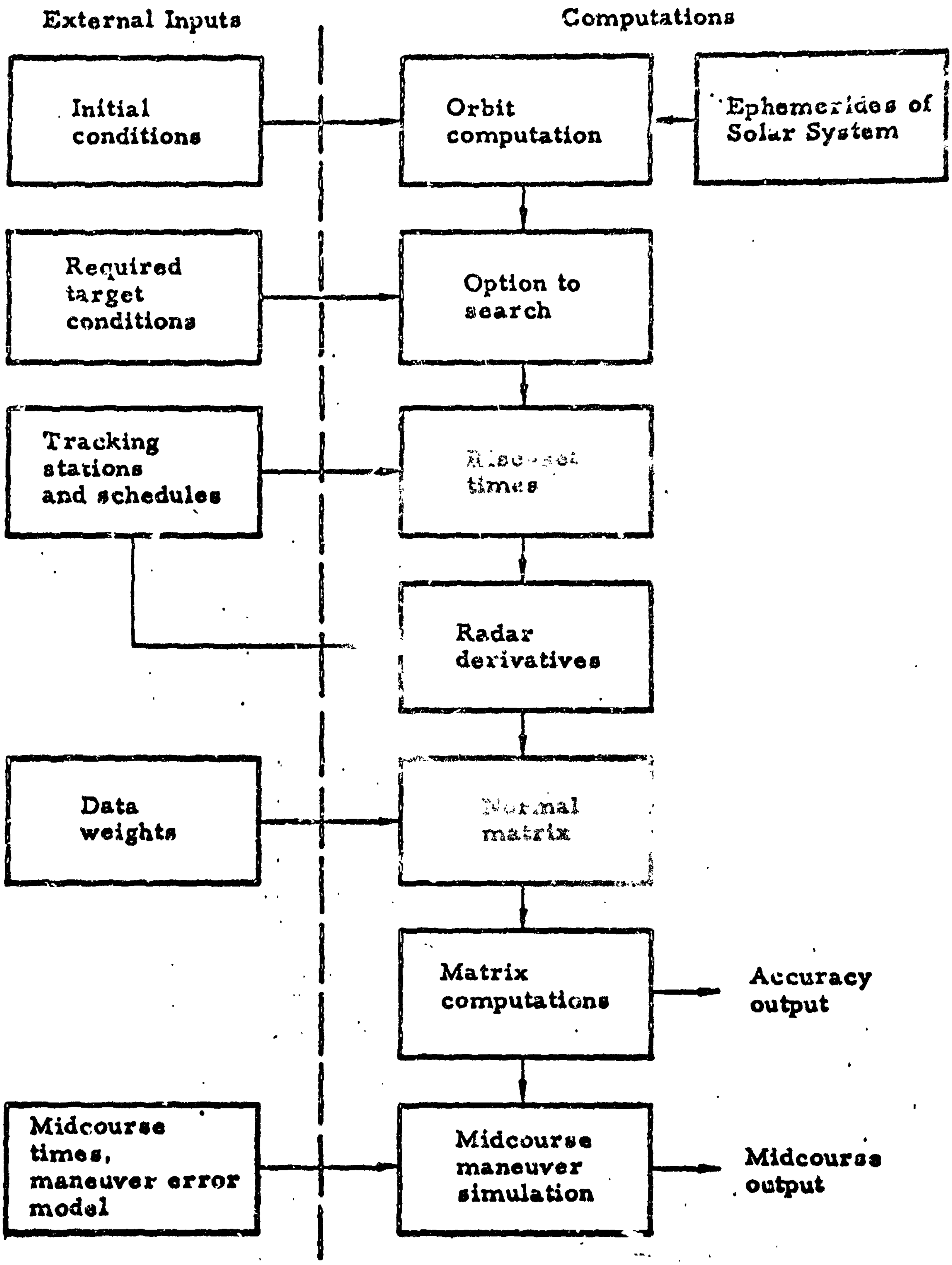

FIGURE 1. General Block Diagram

d) the program may also be used to simulate midcourse maneuver errors from a given set of systems performance parameters and prescribed tracking data.

We shall now describe the functions of the major program blocks:

1) Orbit and Ephemeris Computation

The trajectory of a spacecraft is usually found by solving the equations of motion including all pertinent force terms. To enhance the speed of orbit computation, deviations from Kepler motion are neglected in the present instance, so that all trajectories are conics For cases in which there is a sequence of primary attracting centers, a succession of conics are matched together at the boundaries of the sphere of action for the various bodies.* Such a procedure removes the necessity for any integration of the equations of motion. The computation scheme is given in Appendix 1.

In addition, the ephemerides for the pertinent celestial bodies are computed from Kepler formulas using mean orbital elements which include secular variation terms but not periodic ones. A provision is made to accept osculating elements at a fixed epoch if higher accuracy is required. However, in most instances, the tracking accuracy should not be critically affected by small deviations in the positions of the celestial bodies from their actual position.

2) Search Routine

One required program input is a set of initial orbit conditions yielding approximately the desired final conditions at the target body. These input conditions will normally be obtained from one of the standard lunar or interplanetary trajectory design programs available. To allow for differences in computational models used by TAPP and other programs, a search routine is provided in TAPP to achieve a required set of final conditions. This is accomplished by a differential correction process on the initial conditions. Given an initial vector, x_o, which yields a reasonably close value of the

* See Appendix 6, Page 70.

required final vector b; the routine computes the differential coefficient matrix, $\left|\frac{\partial b}{\partial x_o}\right|$, and finds the correction vector

$$\delta x_o = \left(\frac{\partial b}{\partial x_o}\right)^{-1} \left[b \text{ (required)} - b \text{ (computed)}\right]$$

The new values $x_1' = x_o + \delta x_o$ are used to compute the new values b_1. The process is repeated until the required conditions are achieved. The search may be carried out by varying the injection conditions at the earth or the velocity at infinity on the escape hyperbola. Further details are given in Appendix 5.

3) Rise and Set Times

To insure that the simulated observation times correspond realistically with the given orbit and tracking stations, visibility times from each station are computed over the period of interest.

This is accomplished in the program by computing the elevation angle, E, from the tracker at prescribed intervals over the span of tracking. In particular, we compute

$$\delta = \sin E - \sin \gamma_o = \frac{R_s \cdot \rho}{|R_s||\rho|} - \sin \gamma_o$$

where

R_s is the position vector of the station

ρ is the vector from station to spacecraft

γ_o is the minimum elevation before visibility is said to occur (usually different from zero)

The spacecraft is visible from a given station if $\delta > 0$. The rise-set intervals are found by interpolating for the times at which δ changes sign.

In the case of lunar and deep space vehicles, the spacecraft has a slow angular rate with respect to the earth after the initial day or two. Since the station coordinates have a period of one sidereal day, the rise-set times on the nth day are reasonable first approximations to the rise-set times on the n + 1st day. This fact is used to speed the determination of rise and set times over a long trajectory time span.

In anticipation of lunar satellites, the rise-set routine also finds the occultation times of the spacecraft by the moon. Only visible, non-occulted times are used in simulated observations.

4) Radar Derivatives

The radar derivatives are the regression coefficients appearing in the least squares estimation of x_o. In the present program they are obtained by use of the differentiation chain rule. Let R_i be the ith radar observation; t the time of the observation; x(t) the position and velocity at time t; and x_o the value of x at the epoch, t_o;* then in matrix notation, $\left(\frac{\partial R_i}{\partial x_o}\right)$ is

$$\left(\frac{\partial R_i}{\partial x_o}\right) = \left(\frac{\partial R_i}{\partial x}\right) \left(\frac{\partial x}{\partial x_o}\right)$$

where

$\left(\frac{\partial R_i}{\partial x}\right)$ is the (1 x 6) matrix of derivatives of R_i with respect to x(t). It is obtained directly from the definition of $R_i(x)$ by differentiation.

$\left(\frac{\partial x}{\partial x_o}\right)$ is the (6 x 6) variational matrix for the change $\delta x(t)$ due to an initial increment δx_o.

Since conic formulas are used to approximate the trajectory, $\left(\frac{\partial x}{\partial x_o}\right)$ is obtained by differentiation of the Kepler formulas. The matrix $\left(\frac{\partial x}{\partial x_o}\right)$ is given in Appendix 1 and the quantities $\left(\frac{\partial R_i}{\partial x}\right)$ for the various data types are tabulated in Appendix 3.

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* For illustrative purposes, we are considering x_o to be the initial conditio vector. However, x_o may, in general, include physical constants elements as well.

5a) Formation of the A Matrix

Each row of the A matrix consists of the coefficients of one equation of condition

$$\Delta R = \alpha_1 \Delta a_1 + \alpha_2 \Delta a_2 + \cdots \alpha_n \Delta a_n$$

where the a_i are the variables to be adjusted, (e.g., injection conditions, physical constants, station location, data biases, etc.). The α_i are the coefficients which make up a row of the A matrix. This section deals with the formation of the A matrix when initial conditions, dynamical constants such as mass of the sun, non-dynamical constants, such as speed of light or station location errors, and data biases are to be "adjusted" (i.e., fitted to the equations of conditions in a least square sense). In general,

$$(\alpha) = \left(\frac{\partial R}{\partial x_o}\right) = \left(\frac{\partial R}{\partial x}\right)\left(\frac{\partial x}{\partial x_o}\right)$$

where (α) is now to be considered as a matrix and x_o corresponds to the a_i.

As a specific example, let us consider one case of constructing such an A matrix.

Let the symbol "$\{\ \}$" mean "a set of size".

Let x_o be the initial conditions. $x_o = \{6\}$

Let μ be the other dynamical constants. $\mu = \{p\}$

(e.g., mass of sun, mass of earth)

Let λ be the non-dynamical constants. $\lambda = \{q\}$

(e.g., station uncertainties, speed of light).

Let R be the set of unbiased observations from non-biased station location $R = \{\ell\}$

Let R_B be the set of biased observations from non-biased station locations $R = \{m\}$

Let R_A be the set of non-biased observations from biased station locations $R = \{\theta\}$

Let R_C be the set of biased observations from biased station location $R = \{r\}$

Total number of observations $= n = \ell + m + \theta + r$

Total number of variables $= N = 6 + p + q + m$

The A matrix is then formed by

$$
A = \begin{bmatrix}
\left(\frac{\partial R}{\partial x}\right)_{\ell \times 6} & (0)_{\ell \times q} & (0)_{\ell \times m} \\
\left(\frac{\partial R_B}{\partial x}\right)_{m \times 6} & (0)_{m \times q} & (1)_{m \times m} \\
\left(\frac{\partial R_A}{\partial x}\right)_{\theta \times 6} & \left(\frac{\partial R_A}{\partial \lambda}\right)_{\theta \times q} & (0)_{\theta \times m} \\
\left(\frac{\partial R_C}{\partial x}\right)_{r \times 6} & \left(\frac{\partial R_C}{\partial x}\right)_{r \times q} & (1)_{r \times m}
\end{bmatrix}
\begin{bmatrix}
\left(\frac{\partial x}{\partial x_0}\right)_{6 \times 6} & \left(\frac{\partial x}{\partial \mu}\right)_{6 \times p} & (0)_{6 \times q} & (0) \\
(0)_{q \times 6} & (0)_{q \times p} & {}^{*}(1)_{q \times q} & (0) \\
(0)_{m \times 6} & (0)_{m \times p} & (0)_{m \times q} & {}^{*}(1)
\end{bmatrix}
$$

where * means the identity square matrix.

$$
A = \begin{bmatrix}
\left(\frac{\partial R}{\partial x}\right)\left(\frac{\partial x}{\partial x_0}\right)_{\ell \times 6} & \left(\frac{\partial R}{\partial x}\right)\left(\frac{\partial x}{\partial \mu}\right)_{\ell \times p} & (0)_{\ell \times q} & (0)_{\ell \times m} \\
\left(\frac{\partial R_B}{\partial x}\right)\left(\frac{\partial x}{\partial x_0}\right)_{m \times 6} & \left(\frac{\partial R_B}{\partial x}\right)\left(\frac{\partial x}{\partial \mu}\right)_{m \times p} & (0)_{m \times q} & (1)_{m \times m} \\
\left(\frac{\partial R_A}{\partial x}\right)\left(\frac{\partial x}{\partial x_0}\right)_{\theta \times 6} & \left(\frac{\partial R_A}{\partial x}\right)\left(\frac{\partial x}{\partial \mu}\right)_{\theta \times p} & \left(\frac{\partial R}{\partial \lambda}\right)_{\theta \times q} & (0)_{\theta \times m} \\
\left(\frac{\partial R_C}{\partial x}\right)\left(\frac{\partial x}{\partial x_0}\right)_{r \times 6} & \left(\frac{\partial R_C}{\partial x}\right)\left(\frac{\partial x}{\partial \mu}\right)_{r \times p} & \left(\frac{\partial R_C}{\partial \lambda}\right)_{r \times q} & (1)_{r \times m}
\end{bmatrix}
$$

5b) Normal Matrix

Let R be the vector consisting of all the individual observations, R_i. The elements of the matrix $A = \left(\frac{\partial R}{\partial x_o}\right)$ are formed in accordance with a prescribed set of rules which dictate the type and frequency of the simulated tracking data. The normal matrix is then simply

$$A'WA = \left(\frac{\partial R}{\partial x_o}\right)' W\left(\frac{\partial R}{\partial x_o}\right)$$

where a prime denotes transpose, and W is the diagonal matrix of final weights assigned to the observations. W is also computed in accordance with a set of rules which are given in Appendix 8.

6) Tracking Accuracy Output

The covariance matrix of the impact vector (or some appropriate substitute) is the criterion of tracking accuracy. To elaborate on its computation, we define the notations:

R - m vector of actual observations including noise.

x_{ot} - n vector of true orbital parameters to be estimated.

x_o - initial estimate of x_{ot}.

p - q vector of parameters (usually physical constants) which affect the values of the computed observations but which are not being estimated.

$R = R(x_o, p)$ - m vector of computed values of the observables based on the initial values, x_o and p.

Γ_o - covariance matrix of the initial estimate, x_o.

Λ_p - covariance matrix of the vector p (assumed given).

A - m x n matrix of partial derivatives, $\left(\frac{\partial R}{\partial x_o}\right)$.

P - m x q matrix of partial derivatives, $\left(\frac{\partial R}{\partial p}\right)$.

W - m x m diagonal matrix of final least squares weights.

In most of the following, we absorb $\sqrt{W}$ into the A and P matrices, i. e.

$$A'WA \longrightarrow A'A \qquad \sqrt{W}A \longrightarrow A$$
$$A'WP \longrightarrow A'P \qquad \sqrt{W}P \longrightarrow P$$

In performing the least squares fit, we hold the vector p fixed but include the effects of its uncertainty in computing the covariance matrix of impact errors. In general, the p vector will include quantities such as mass of the earth, moon, station location, velocity of light, etc. The errors in x_o, p, and the noise on the observations are assumed to be independent of each other for the present.

If the assumed values of p coincide with the true values, p_t, then the least squares estimate of x_{ot} is the value x_{ls} which minimizes the weighted sum

$$S = \left[\hat{R} - R\,(x_o, p_t) - A\,(x_{ls} - x_o)\right]'\; W\left[\hat{R} - R\,(x_o, p_t) - A\,(x_{ls} - x_o\right.$$

If in addition x_o is an a priori estimate of x_{ot} with covariance matrix Γ_o, then the combined least squares and a priori estimate, $\tilde{x}_o$, is obtained from the equation*

$$\tilde{x}_o = x_o + K\,A'\left[\hat{R} - R\,(x_o, p_t)\right] \qquad (1)$$

$$K = (A'A + \Gamma_o^{-1})^{-1}$$

* See footnote on next page

A small increment $\delta p = p - p_t$ will yield a slightly different estimate, $\hat{x}_o$, where

$$\hat{x}_o = x_o + KA' \left[\hat{R} - R(x_o, p) - P\,\delta p \right] \tag{2}$$

If $\delta\hat{x}_o = \hat{x}_o - x_{ot}$ and $\delta x_o = x_o - x_{ot}$, we obtain from (2)

$$\begin{aligned} \delta\hat{x}_o &= \delta x_o + KA' \left[\hat{R} - R(x_{ot}, p_t) - A\,\delta x_o - P\,\delta p \right] \\ &= K\left[A'\,\delta R - A'P\,\delta p + \Gamma_o^{-1}\,\delta x_o \right] \end{aligned} \tag{3}$$

* Footnote from previous page :

If $\Gamma_o^{-1} = 0$ (no a priori knowledge), the estimate of x_{ot} reduces to x_{ls} where

$$x_{ls} = x_o + (A'A)^{-1} A' \left[R - R(x_o, p_t) \right] \tag{1a}$$

which is the usual formula based on a least squares criterion. Equation (1) combines the a priori estimate with the least squares estimate, all into one operation. As shown in Reference [1], it is equivalent to finding the least squares fit as in (1a), and then combining with the a priori estimate in accordance with the formula

$$\tilde{x}_o = \left[\Lambda_{ls}^{-1} + \Gamma_o^{-1} \right]^{-1} \left[\Lambda_{ls}^{-1} x_{ls} + \Gamma_o^{-1} x_o \right] \tag{1b}$$

where Λ_{ls} is the covariance matrix of x_{ls}. The methods are equivalent and the covariance matrix of $\tilde{x}_o$ is given by the first term of (4) if one assumes that there are no errors in p.

The covariance matrix of the estimate is

$$\Lambda_o = \overline{\delta\hat{x}_o \delta\hat{x}_o'} = K\left[J + A' P\Lambda_p P'A\right] K \tag{4}$$

$$J = A' NA + \Gamma_o^{-1} \tag{5}$$

where the bar denotes an ensemble average and Λ_p is the a priori covariance matrix of p; N is the product of the diagonal matrix of the variances on the noise and the weighting matrix W. The matrix, A, always has included in it the factor $\sqrt{W}$; otherwise an additional factor of W would appear in (5).

The differential errors in the impact vector, $b = b\ (x_o,\ p)$ are related linearly to $\delta\hat{x}_o$ and δp. We have

$$\delta b = \lambda\, \delta\hat{x}_o + \mu\delta p$$

where

$$\lambda = \frac{\partial b}{\partial x_o}$$

$$\mu = \frac{\partial b}{\partial p}$$

$$\nu = -KA'P$$

The covariance matrix of b is

$$\Lambda_b = \overline{\delta b\ \delta b'} = \overline{(\lambda\, \delta\hat{x}_o + \mu\delta p)\ (\lambda\, \delta\hat{x}_o + \mu\delta p)'}$$

$$= \lambda\, KJK\lambda' + (\mu + \lambda\nu)\ \Lambda_p\ (\mu + \lambda\nu)' \tag{6}$$

λ and μ are the usual explicit partial derivatives of b with respect to x_o and p respectively. $\delta b = \lambda\nu\delta p$ is an additional error term in b due to an error in $\hat{x}_o$ arising from an incremental change δp.

Equation (4) is the formula for finding Λ_b when δx_o and $\delta\dot{p}$ are independent. Since Λ_b is the criterion which measures the tracking accuracy, much of the remainder of this write-up deals with the details and options pertaining to its computation from hypothetical observations.

In Appendix 4, b is shown to be a vector consisting of the two components of the miss vector, $m = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$ and the total flight time, t_f.

$$b = \begin{bmatrix} m_1 \\ m_2 \\ t_f \end{bmatrix}$$

m_1 and m_2 define a plane which will be called the impact parameter plane. $\Lambda_b = \overline{\delta b\, \delta b'}$ is a 3 x 3 matrix whose upper left hand 2 x 2 is given by $\Lambda_m = \overline{\delta m\, \delta m'}$. We rewrite this as

$$\Lambda_m = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \qquad (7)$$

The quadratic form $\delta m' \Lambda_m^{-1} \delta m$ = constant describes a dispersion ellipse of constant probability in the (m_1, m_2) plane. Λ_m may be diagonalized by means of an orthogonal transformation to new variables M where

$$M = Um$$

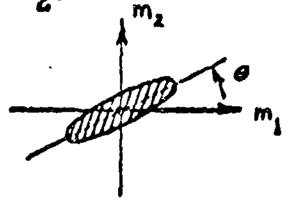
so that

$$\Lambda_M = U\Lambda_m U' = \begin{pmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{pmatrix} \quad \text{and,} \qquad (8)$$

$$\lambda_{1,2}^2 = \frac{\sigma_1^2 + \sigma_2^2 \pm \quad (\sigma_1^2 - \sigma_2^2)^2 - (1 - \rho^2)}{2}$$

U is a rotation from the m_1 axis to the major axis of the dispersion ellipse. The angle of rotation is θ where (assume $\sigma_1 > \sigma_2$)

$$\theta = \frac{1}{2} \tan^{-1} \frac{2\rho\sigma_1\sigma_2}{\sigma_1^2 - \sigma_2^2} \qquad (9)$$



The quantities Λ_b, Λ_M, and θ are computed at various stages of the simulation as an indication of the tracking accuracy.

7) Midcourse Maneuvers

Another use of the program is to simulate the errors of a midcourse velocity correction. This is done by supplementing Λ_b with an error covariance matrix due to the imperfect execution of the maneuver. Since the maneuver system errors and the tracking noise are assumed to be independent, the covariance matrices from the two sources add directly. The program may be instructed to perform either a hypothetical or an actual maneuver.

The difference between them is that in the first case the correction velocity errors are not propagated into the future. Their effects on Λ_b are computed at the time of the hypothetical maneuver and are dropped for further calculations. The object is to display the effects on Λ_b of the maneuver errors at various points along the orbit as the amount of tracking and the error coefficients vary in time.

In contrast, the simulation of an actual midcourse implies that the maneuver errors are permanently implanted in the orbit as they always are in real life. All computations of Λ_b after the maneuver will have included in them the errors arising from the performance of the maneuver. In both cases we assume that the mean of the midcourse velocity magnitude is zero so that the orbit remains unaltered from the nominal even though the errors are added on. This is a valid procedure if everything is linear, i.e. the error coefficients do not change rapidly in the vicinity of the nominal. The program may be required to perform a series of hypothetical maneuvers but only one actual one at this time.*

a) Hypothetical Maneuvers (abbreviated hm)

Usually, a sequence of hypothetical maneuvers are called for along an orbit. To illustrate the effect at the ith point, use the symbols:

Λ_{bi} - covariance matrix of b just prior to the ith hm.

Λ_{ai} - covariance matrix of b just after the ith hm.

Λ_e - covariance matrix of velocity errors due to imperfect execution of the required maneuver. See Appendix 7.

Λ_e is a 3 x 3 matrix but may be used as a partitioned 6 x 6 matrix $\Lambda_e = \begin{pmatrix} 0 & 0 \\ 0 & \Lambda_e \end{pmatrix}$.

x_i = (r_i, v_i) - the spacecraft position and velocity vector with respect to the force center at t_i, the time of the ith hm.

Λ_i = covariance matrix of x_i due to tracking

x_o - position and velocity at the initial epoch, t_o, of the phase during which the maneuver occurs

Λ_o = covariance matrix of x_o due to tracking only

A straightforward way of computing Λ_{bi} is to update the epoch to the ith hm; compute Λ_i due to tracking; Λ_{bi} is just $\Lambda_{bi} = \left(\frac{\partial b}{\partial x_i}\right) \Lambda_i \left(\frac{\partial b}{\partial x_i}\right)'$.

* TAPP Mod II has a multiple midcourse maneuver capability.

Λ_{ai} is found by replacing Λ_i by $\Lambda_i + \Lambda_e$ in Λ_{bi}. However, frequent updating involves some tedious matrix manipulations if physical constants are involved. A somewhat simpler scheme is used for hm's in the present program by keeping the epoch at t_o. At t_i compute

$$\Lambda_{oi} = \left(\frac{\partial x_o}{\partial v_i}\right) \Lambda_e \left(\frac{\partial x_o}{\partial v_i}\right)'$$

after which

$$\Lambda_{ai} = \Lambda_{bi} + \left(\frac{\partial b}{\partial x_o}\right) \Lambda_{oi} \left(\frac{\partial b}{\partial x_o}\right)' = \left(\frac{\partial b}{\partial x_o}\right) (\Lambda_o + \Lambda_{oi}) \left(\frac{\partial b}{\partial x_o}\right)'$$

(neglecting physical constant errors in this case)

For computational purposes, $\left(\frac{\partial x_o}{\partial v_i}\right)$ is obtained from $\left(\frac{\partial x_i}{\partial x_o}\right)^{-1}$ in which $x_i = (r_i, v_i)$ i.e.

$$\left(\frac{\partial x_o}{\partial x_i}\right) = \left(\frac{\partial x_i}{\partial x_o}\right)^{-1} = \begin{bmatrix} \dfrac{\partial x_o}{\partial r_i} \\ \hline \dfrac{\partial x_o}{\partial v_i} \end{bmatrix}$$

b) <u>Actual Maneuvers</u>

After an actual maneuver, the epoch is moved to the time of the maneuver. Calling the new point (t_o, x_o) with a priori covariance matrix Λ_o, then

$$\Lambda_a = \left(\frac{\partial b}{\partial x_o}\right) (\Lambda_o + \Lambda_e) \left(\frac{\partial b}{\partial x_o}\right)'$$

If more tracking data is added after t_o, then the effects of the new data can be incorporated into the covariance matrix of x_o by the formula

$$\tilde{\Lambda}_o = K J K$$

where

$$K = \left[(\Lambda_o + \Lambda_e)^{-1} + A_1' A_1 \right]^{-1}$$

$$J = A_1' N_1 A_1 + (\Lambda_o + \Lambda_e)^{-1}$$

A_1 and N_1 are quantities referring to the new data having meanings which correspond to A and N in (5). The covariance matrix for b is then simply

$$\Lambda_b = \left(\frac{\partial b}{\partial x_o} \right) \tilde{\Lambda}_o \left(\frac{\partial b}{\partial x_o} \right)'$$

If physical constant uncertainties are considered, the situation becomes more involved and will be dealt with in Appendix 6.

APPENDIX 1 ORBIT COMPUTATION AND V TIONAL EQUATIONS

This appendix tabulates some expr...ons for the derivatives of position and velocity on Keplerian conics with respect to the initial conditions. Of the many sets of parameters which may be used to represent the orbit, the set chosen is the one given by the initial cartesian components of position and velocity of the point mass with respect to the dynamical center.

The method of representing the orbit is completely general in that it includes elliptic, parabolic, hyperbolic, circular, and zero inclination orbits with almost no alterations in the computation procedure. There is no singularity in the transition from the elliptic to the hyperbolic case so that the expressions for the derivatives are valid even in the limit of the parabolic case. The usual transcendental functions are replaced by similar power series which terminate with a single term in the parabolic limit.

Derivatives for transforming covariance matrices to the usual classical elements and the polar coordinates are also tabulated because these coordinate systems are often useful in guidance analysis.

Let $\bar{r}$ (x, y, z) be the position vector of the point mass in an inertial, cartesian frame with respect to the dynamical center;

$\bar{v}$ $(\dot{x}, \dot{y}, \dot{z})$ the velocity vector in the same frame.

The notation x appearing in a matrix is taken to mean the combined vector

$x = (\bar{r}, \bar{v})$

I. Orbit Computation

Given the initial values $\bar{r}_o = (x_o, y_o, z_o)$ and $\bar{v}_o$ $(\dot{x}_o, \dot{y}_o, \dot{z}_o)$ at $t = t_o$; the position $\bar{r}(t)$ and velocity $\bar{v}(t)$ is found from the outline below. The derivation of the orbit formulas may be found in Reference [2]. The constants of the motion are:

$$r_o = |\bar{r}_o| , \qquad v_o = |\bar{v}_o|$$

$$d = \frac{\bar{r}_o \cdot \bar{v}_o}{\sqrt{\mu}}$$

$$\lambda = \frac{r_o v_o^2}{\mu}$$

$$c_o = \lambda - 1$$

$$\frac{1}{a} = \frac{2 - \lambda}{r_o}$$

where μ is the mass constant of the central body.

Kepler's Equation consists of the three simultaneous equations

$$M = \sqrt{\mu}\,(t - t_o) = r_o\, \Theta + dc + c_o\, u \tag{1.1}$$

$$u = \frac{\Theta^3}{3!} - \frac{\Theta^5}{a5!} + \frac{\Theta^7}{a^2 7!} \cdots \tag{1.2}$$

$$c = \frac{\Theta^2}{2} - \frac{\Theta^4}{a4!} + \frac{\Theta^6}{a^2 6!} \cdots \tag{1.3}$$

which for given $t - t_o$ are to be solved by an iterative process for Θ, u, and c. In the present program Θ is used as the independent variable instead of t and M, u, and c are evaluated explicitly.

Next compute

$$s = \theta - \frac{u}{a} \tag{1.4}$$

$$r = r_o + ds + c_o\, c \tag{1.5}$$

$$f = 1 - \frac{c}{r_o} \tag{1.6}$$

$$g = \frac{M - u}{\sqrt{\mu}} \tag{1.7}$$

$$\dot{f} = \frac{-\sqrt{\mu}\, s}{\mu\, r_o} \tag{1.8}$$

$$\dot{g} = 1 - \frac{c}{r} \tag{1.9}$$

Finally

$$\bar{r}(t) = f\, \bar{r}_o + g\, \bar{v}_o \tag{1.10}$$

$$\bar{v}(t) = \dot{f}\, \bar{r}_o + \dot{g}\, \bar{v}_o \tag{1.11}$$

II. Partial Derivative Matrix

The variational matrix $\left(\frac{\partial x}{\partial x_o}\right)$ may now be computed by direct differentiation of the above formulas.

We begin by defining ξ and η by

$$\xi = \frac{\partial u}{\partial\left(\frac{1}{a}\right)} = -\frac{\theta^5}{5!} + \frac{2}{a}\,\frac{\theta^7}{7!} - \frac{3}{a^2}\,\frac{\theta^9}{9!} + \ldots \tag{1.12}$$

$$\eta = \frac{\partial c}{\partial\left(\frac{1}{a}\right)} = -\frac{\theta^4}{4!} + \frac{2}{a}\,\frac{\theta^6}{6!} + \frac{3}{a^2}\,\frac{\theta^8}{8!} \ldots \tag{1.13}$$

Differentiation of (1.1) with the aid of (1.5) yields

$$r \frac{\partial \theta}{\partial x_o} = \left[\frac{2}{r_o^2} \quad (d\eta + c_o \ \xi) - \theta - \frac{u\lambda}{r_o} \right] \frac{x_o}{r_o} \quad - \frac{c\dot{x}_o}{\sqrt{\mu}}$$

$$r \frac{\partial \theta}{\partial \dot{x}_o} = \left[d\eta + c_o \ \xi - u \ r_o \right] \frac{2\dot{x}_o}{\mu} - \frac{cx_o}{\sqrt{\mu}}$$

Differentiating (1.2 - 1.5) in order yields

$$\frac{\partial u}{\partial x_o} = c \frac{\partial \theta}{\partial x_o} - 2 \ \xi \ \frac{x_o}{r_o^3}$$

$$\frac{\partial u}{\partial \dot{x}_o} = c \frac{\partial \theta}{\partial \dot{x}_o} - 2 \ \xi \ \frac{\dot{x}_o}{\mu}$$

$$\frac{\partial c}{\partial x_o} = s \frac{\partial \theta}{\partial x_o} - 2 \ \eta \ \frac{x_o}{r_o^3}$$

$$\frac{\partial c}{\partial \dot{x}_o} = s \frac{\partial \theta}{\partial \dot{x}_o} - 2 \ \eta \ \frac{\dot{x}_o}{\mu}$$

$$\frac{\partial s}{\partial x_o} = \frac{\partial \theta}{\partial x_o} + 2 \ u \ \frac{x_o}{r_o^3} - \frac{1}{a} \frac{\partial u}{\partial x_o}$$

$$\frac{\partial s}{\partial \dot{x}_o} = \frac{\partial \theta}{\partial \dot{x}_o} + \frac{2u \ \dot{x}_o}{\mu} - \frac{1}{a} \frac{\partial u}{\partial \dot{x}_o}$$

$$\frac{\partial r}{\partial x_o} = \frac{s\dot{x}_o}{\sqrt{\mu}} + d\,\frac{\partial s}{\partial x_o} + c_o\,\frac{\partial c}{\partial x_o} + \frac{x_o}{r_o}\left(1 + \frac{c\lambda}{r_o}\right)$$

$$\frac{\partial r}{\partial \dot{x}_o} = \frac{s x_o}{\sqrt{\mu}} + d\,\frac{\partial s}{\partial \dot{x}_o} + c_o\,\frac{\partial c}{\partial \dot{x}_o} + \frac{2\, r_o\, c\, \dot{x}_o}{\mu}$$

Next, from (1.6 - 1.9) we obtain

$$\frac{\partial f}{\partial x_o} = -\frac{\partial c}{\partial x_o}\,\frac{1}{r_o} + \frac{c x_o}{r_o^{\,3}}$$

$$\frac{\partial f}{\partial \dot{x}_o} = -\frac{1}{r_o}\,\frac{\partial c}{\partial \dot{x}_o}$$

$$\frac{\partial g}{\partial x_o} = -\frac{\partial u}{\partial x_o}\,\frac{1}{\sqrt{\mu}}$$

$$\frac{\partial g}{\partial \dot{x}_o} = -\frac{\partial u}{\partial \dot{x}_o}\,\frac{1}{\sqrt{\mu}}$$

$$\frac{\partial \dot{f}}{\partial x_o} = -\frac{\sqrt{\mu}}{r}\left[\frac{1}{r_o}\,\frac{\partial s}{\partial x_o} - \frac{s}{r\, r_o}\,\frac{\partial r}{\partial x_o} - \frac{x_o\, s}{r_o^{\,3}}\right]$$

$$\frac{\partial \dot{f}}{\partial \dot{x}_o} = -\frac{\sqrt{\mu}}{r}\left[\frac{1}{r_o}\,\frac{\partial s}{\partial \dot{x}_o} - \frac{s}{r\, r_o}\,\frac{\partial r}{\partial \dot{x}_o}\right]$$

$$\frac{\partial \dot{g}}{\partial x_o} = -\frac{1}{r}\left[\frac{\partial c}{\partial x_o} + \frac{c}{r}\,\frac{\partial r}{\partial x_o}\right]$$

$$\frac{\partial \dot{g}}{\partial \dot{x}_o} = -\frac{1}{r}\left[\frac{\partial c}{\partial \dot{x}_o} + \frac{c}{r}\,\frac{\partial r}{\partial \dot{x}_o}\right]$$

Finally assembling it all in (1.9), (1.10)

$$\frac{\partial \bar{r}}{\partial x_0} = \frac{\partial f}{\partial x_0} \bar{r}_0 + \frac{\partial g}{\partial x_0} \bar{v}_0 + f \frac{\partial \bar{r}_0}{\partial x_0}$$

$$\frac{\partial \bar{r}}{\partial \dot{x}_0} = \frac{\partial f}{\partial \dot{x}_0} \bar{r}_0 + \frac{\partial g}{\partial \dot{x}_0} \bar{v}_0 + g \frac{\partial \bar{v}_0}{\partial \dot{x}_0}$$

$$\frac{\partial \bar{v}}{\partial x_0} = \frac{\partial \dot{f}}{\partial x_0} \bar{r}_0 + \frac{\partial \dot{g}}{\partial x_0} \bar{v}_0 + \dot{f} \frac{\partial \bar{r}}{\partial x_0}$$

$$\frac{\partial \bar{v}}{\partial \dot{x}_0} = \frac{\partial \dot{f}}{\partial \dot{x}_0} \bar{r}_0 + \frac{\partial \dot{g}}{\partial \dot{x}_0} \bar{v}_0 + \dot{g} \frac{\partial \bar{v}_0}{\partial \dot{x}_0}$$

Derivatives with respect to y, z are obtained by replacing x by y, and z respectively above.

Derivatives with respect to the gravitational constant, μ, are found as before by differentiation. The final results yield derivatives in terms of percentage change $\frac{\delta\mu}{\mu}$. We have:

$$\mu \frac{\partial \theta}{\partial \mu} = \frac{1}{2r} (M + cd) + \frac{\lambda}{r\, r_0} (u\, r_0 - c_0\, \xi - d\eta)$$

$$\mu \frac{\partial u}{\partial \mu} = c\mu \frac{\partial \theta}{\partial \mu} + \frac{\lambda \xi}{r_0}$$

$$\mu \frac{\partial c}{\partial \mu} = s\mu \frac{\partial \theta}{\partial \mu} + \frac{\lambda \eta}{r_0}$$

$$\mu \frac{\partial s}{\partial \mu} = \mu \frac{\partial \theta}{\partial \mu} - \frac{\mu}{a} \frac{\partial u}{\partial \mu} - \frac{\lambda u}{r_o}$$

$$\mu \frac{\partial r}{\partial \mu} = \mu d \frac{\partial s}{\partial \mu} + \mu c_o \frac{\partial c}{\partial \mu} - \frac{sd}{2} - c\lambda$$

$$\mu \frac{\partial f}{\partial \mu} = - \frac{\mu}{r_o} \frac{\partial c}{\partial \mu}$$

$$\mu \frac{\partial \dot{f}}{\partial \mu} = - \frac{\mu^{3/2}}{r\, r_o} \frac{\partial s}{\partial \mu} - \dot{f} \left[\frac{\mu}{r} \frac{\partial r}{\partial \mu} - \frac{1}{2} \right]$$

$$\mu \frac{\partial g}{\partial \mu} = -\sqrt{\mu} \frac{\partial u}{\partial \mu} + \frac{1}{2} \frac{u}{\sqrt{\mu}}$$

$$\mu \frac{\partial \dot{g}}{\partial \mu} = \frac{\mu}{r} \left(\frac{c}{r} \frac{\partial r}{\partial \mu} - \frac{\partial c}{\partial \mu} \right)$$

Finally,

$$\mu \frac{\partial \bar{r}}{\partial \mu} = \mu \frac{\partial f}{\partial \mu} \bar{r}_o + \mu \frac{\partial g}{\partial \mu} \bar{v}_o$$

$$\mu \frac{\partial \bar{v}}{\partial \mu} = \mu \frac{\partial \dot{f}}{\partial \mu} \bar{r}_o + \mu \frac{\partial \dot{g}}{\partial \mu} \bar{v}_o$$

As pointed out at the beginning; the previous formulas are valid for any type of conic. However, one disadvantage of the method is that the series expressions, u, c, ξ, η converge rather slowly as the argument becomes large. While one can always evaluate these expressions by including more and more terms, the amount of labor becomes excessive after a short while.

One way of overcoming this is to move the initial epoch along as one proceeds in the orbit so that the argument, $\theta = \sqrt{a}\,(E - E_0)$, in the series is always small. A second way is to classify orbits according to the value of a and use the equivalent closed expressions when $|a|$ is small. The latter method was adopted for our purposes since frequent changes of epoch involve considerable matrix manipulations when tracking is involved. This classification is in no way restrictive since there is no singularity at the transitional values of a.

There are six quantities which are altered depending on the value of a. These quantities and their equivalent closed expressions are given in Table 1.

III. Auxiliary Derivatives

If the set of quantities $x_0 = (\bar{r}_0, \bar{v}_0)$ is estimated from noisy data, the transformation of the resultant covariance matrix, $C\,(x_0)$, to a new set of variables, w, is given by

$$C\,(w) = \left(\frac{\partial w}{\partial x_0}\right) C\,(x_0) \left(\frac{\partial w}{\partial x_0}\right)'$$

(prime denotes transpose)

The sets which are frequently of interest are the polar coordinates $v_0 = (r\ \alpha\ \delta\ v\ \beta\ A)_{t\,=\,t_0}$ defined below and the set of classical elements $a = (a\ e\ i\ \Omega\ \omega\ M_0)$. We need to find $\left(\frac{\partial v_0}{\partial x_0}\right)$ and $\left(\frac{\partial a}{\partial x_0}\right)$ as well as $\left(\frac{\partial a}{\partial \mu}\right)$.

Variable	1 $0 < a \le a_1$	2 $a_1 < \lvert a \rvert < \infty$	3 $-a_1 \le a \le 0$
θ	$\lvert a \rvert^{1/2} \Delta E$	θ	$\lvert a \rvert^{1/2} \Delta E$
u	$a^{3/2} (\Delta E - \sin \Delta E)$	series	$\lvert a \rvert^{1/2}\, a\, (\Delta E - \sinh \Delta E)$
c	$a\,(1 - \cos \Delta E)$	series	$a\,(1 - \cosh \Delta E)$
s	$\sqrt{a} \sin \Delta E$	series	$\sqrt{\lvert a \rvert} \sinh \Delta E$
ξ	$-a^{5/2}\left[\Delta E \left(1 + \frac{\cos \Delta E}{2}\right) - \frac{3}{2} \sin \Delta E\right]$	series	$-\lvert a \rvert^{5/2}\left[\Delta E \left(1 + \frac{\cosh \Delta E}{2}\right) - \frac{3}{2} \sinh \Delta E\right]$
η	$a^2 \left(\frac{\Delta E}{2} \sin \Delta E + \cos \Delta E - 1\right)$	series	$a^2 \left(- \frac{\Delta E}{2} \sinh \Delta E + \cosh \Delta E - 1\right)$

Note that ΔE is the argument for the closed forms and θ is a derived quantity while for the series θ is the independent argument.

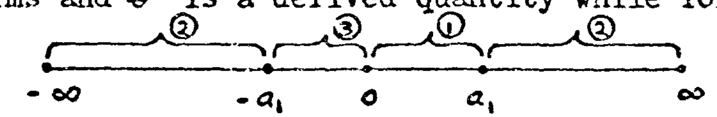


TABLE 1

Table of Variables Whose Computational Formula

Depends on the Value of a

$\left(\frac{\partial v_o}{\partial x_o}\right)$ may be found from differentiating the t formation equations below.
The subscript "o" will be omitted for conver e since the relations hold for all t.

$$r = (x^2 + y^2 + z^2)^{1/2} \qquad = \text{geoc . c distance}$$

$$\alpha = \tan^{-1} y/x \qquad = \text{right ascension}$$

$$\delta = \sin^{-1} z/r \qquad = \text{declination}$$

$$v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} \qquad = \text{velocity magnitude}$$

$$\beta = \cos^{-1} \frac{x\dot{x} + y\dot{y} + z\dot{z}}{rv} \qquad = \text{inclination of the velocity from vertical}$$

$$A = \tan^{-1} \frac{x\dot{y} - y\dot{x}}{r\dot{z} - z\dot{r}} \qquad = \text{azimuth of } v \text{ from north}$$

$$\therefore, \quad \frac{\partial r}{\partial x} = \frac{x}{r} \qquad \frac{\partial r}{\partial y} = \frac{y}{r} \qquad \frac{\partial r}{\partial z} = \frac{z}{r}$$

$$\frac{\partial \alpha}{\partial x} = \frac{-y}{x^2 + y^2}$$

$$\frac{\partial \alpha}{\partial y} = \frac{x}{x^2 + y^2}$$

$$\frac{\partial \delta}{\partial x} = \frac{-xz}{r^2 (x^2 + y^2)^{1/2}}$$

$$\frac{\partial \delta}{\partial y} = \frac{-yz}{r^2 \, (x^2 + y^2)^{1/2}}$$

$$\frac{\partial \delta}{\partial z} = \frac{\sqrt{x^2 + y^2}}{r^2}$$

$$\frac{\partial v}{\partial \dot{x}} = \frac{\dot{x}}{v} , \qquad \frac{\partial v}{\partial \dot{y}} = \frac{\dot{y}}{v} , \qquad \frac{\partial v}{\partial \dot{z}} = \frac{\dot{z}}{v}$$

$$\frac{\partial \beta}{\partial x} = \frac{1}{r v_h} \left[\frac{x \dot{r}}{r} - \dot{x} \right] , \qquad r\dot{r} = x\dot{x} + y\dot{y} + z\dot{z}$$

$$v_h = \sqrt{v^2 - \dot{r}^2}$$

$$\frac{\partial \beta}{\partial y} = \frac{1}{r v_h} \left[\frac{y \dot{r}}{r} - \dot{y} \right]$$

$$\frac{\partial \beta}{\partial z} = \frac{1}{r v_h} \left[\frac{z \dot{r}}{r} - \dot{z} \right]$$

$$\frac{\partial \beta}{\partial \dot{x}} = \frac{1}{r v_h} \left[\frac{\dot{x} \; r \; \dot{r}}{v^2} - x \right]$$

$$\frac{\partial \beta}{\partial \dot{y}} = \frac{1}{r v_h} \left[\frac{\dot{y} \; r \; \dot{r}}{v^2} - y \right]$$

$$\frac{\partial \beta}{\partial \dot{z}} = \frac{1}{r v_h} \left[\frac{\dot{z} \; r \; \dot{r}}{v^2} - z \right]$$

$$\frac{\partial A}{\partial x} = \frac{1}{v_h^2 (x^2 + y^2)} \left[\dot{y}(r\dot{z} - z\dot{r}) - \frac{(x\dot{y} - y\dot{x})}{r}\left(x\dot{z} - z\dot{x} + xz\frac{\dot{r}}{r}\right)\right]$$

$$\frac{\partial A}{\partial y} = \frac{-1}{v_h^2 (x^2 + y^2)} \left[\dot{x}(r\dot{z} - z\dot{r}) + \frac{x\dot{y} - y\dot{x}}{r}\left(y\dot{z} - z\dot{y} + yz\frac{\dot{r}}{r}\right)\right]$$

$$\frac{\partial A}{\partial z} = \frac{\dot{r}(x\dot{y} - y\dot{x})}{r^2 v_h^2}$$

$$\frac{\partial A}{\partial \dot{x}} = \frac{\dot{y}z - \dot{z}y}{r v_h^2}$$

$$\frac{\partial A}{\partial \dot{y}} = \frac{\dot{z}x - \dot{x}z}{r v_h^2}$$

$$\frac{\partial A}{\partial \dot{z}} = -\frac{r}{\dot{r}}\frac{\partial A}{\partial z}$$

The transformation to the classical orbital elements requires the matrix, $\left(\frac{\partial a}{\partial x_o}\right)$. Since the elements are more simply expressed in terms of the polar coordinates, we find $\left(\frac{\partial a}{\partial x_o}\right)$ from

$$\left(\frac{\partial a}{\partial x_o}\right) = \left(\frac{\partial a}{\partial v_o}\right)\left(\frac{\partial v_o}{\partial x_o}\right)$$

$\left(\frac{\partial v_o}{\partial x_o}\right)$ was given above. In terms of the polar coordinates the elements are: (again leaving off the subscripts "o" with the understanding that all the quantities below refer to a common fixed time)

$$a = \frac{r}{2 - \lambda}$$ = major semi-axis

$$e = \left[1 - \lambda (2 - \lambda) \sin^2\beta\right]^{1/2}$$ = eccentricity

$$i = \cos^{-1} (\cos \delta \sin A)$$ = inclination

$$\tan (\alpha - \Omega) = \sin \delta \tan A$$ Ω = ascending node

$$\omega = u - f$$ = argument of perigee

$$\tan f = \sqrt{\frac{p}{\mu}} \frac{r\dot{r}}{p - r}$$

$$p = r\lambda \sin^2\beta$$

$$\tan u = \frac{\sin \delta}{\cos A \cos \delta}$$

$$M = E - e \sin E$$ = mean anomaly

$$r = a (1 - e \cos E)$$

Differentiating, we find

$$\frac{\partial a}{\partial r} = \frac{2}{(2 - \lambda)^2}$$ $\lambda \neq 2$

$$\frac{\partial a}{\partial v} = \frac{\lambda r}{v} \frac{\partial a}{\partial r}$$

$$\frac{\partial e}{\partial r} = \frac{p (\lambda - 1)}{r^2 e}$$ $e \neq o$

$p = a (1 - e^2)$

$$\frac{\partial e}{\partial v} = \frac{2r}{v} \left|\frac{\partial e}{\partial r}\right|$$

$$\frac{\partial e}{\partial \beta} = -\frac{y_\omega}{a} \qquad y_\omega = \frac{r\lambda}{e} \sin\beta \cos\beta$$

$$\frac{\partial i}{\partial \delta} = \sin(\alpha - \Omega)$$

$$\frac{\partial i}{\partial A} = -\cos\delta \cos(\alpha - \Omega)$$

$$\frac{\partial \Omega}{\partial \delta} = -\frac{\cos(\alpha - \Omega)}{\tan i} \qquad i \neq 0$$

$$\frac{\partial \Omega}{\partial A} = -\frac{\sin\delta}{\sin^2 i}$$

$$\frac{\partial \omega}{\partial r} = \frac{y_\omega}{er^2}$$

$$\frac{\partial \omega}{\partial \delta} = \frac{\cos^2(\alpha - \Omega)}{\cos A}$$

$$\frac{\partial \omega}{\partial v} = \frac{2y_\omega}{r\,e\,v}$$

$$\frac{\partial \omega}{\partial \beta} = 2 + \frac{x_\omega}{ae}, \qquad x_\omega = \frac{p - r}{e}$$

$$\frac{\partial \omega}{\partial A} = \frac{\cos\delta \sin(\alpha - \Omega)}{\sin i}$$

$$\frac{\partial M}{\partial r} = \frac{-y_\omega}{\sqrt{|a|\,p}} \left(\frac{p + e^2 r}{r^2 e}\right)$$

$$\frac{\partial M}{\partial v} = \frac{2r}{v} \quad \frac{\partial M}{\partial r}$$

$$\frac{\partial M}{\partial \beta} = -\sqrt{|a|}\ p \ \frac{x_\omega}{ea^2}$$

Some obvious restrictions for $\left|\frac{\partial a}{\partial v_o}\right|$ are noted above, i.e., $(i, e, \frac{1}{a}) \neq 0$.

This simply removes circular and parabolic orbits as well as zero inclination orbit planes. The derivatives remain valid for hyperbolic cases with no change. For the period, apogee, and perigee, we have

$$P = \frac{2\pi a^{3/2}}{\sqrt{\mu}} \qquad = \text{period}$$

$$r_a = a\,(1 + e) \qquad = \text{apogee distance}$$

$$r_p = a\,(1 - e) \qquad = \text{perigee distance}$$

$$\frac{\partial P}{\partial a} = \frac{3}{2} \ \frac{P}{a}$$

$$\frac{\partial r_a}{\partial a} = \frac{r_a}{a} \qquad \frac{\partial r_p}{\partial a} = \frac{r_p}{a}$$

$$\frac{\partial r_a}{\partial e} = a \qquad \frac{\partial r_p}{\partial e} = -a$$

APPENDIX 2 C ATION OF HEMERIS

The positions of th estial bodi e obtained by a series expansion in powers of th ntricity heir mean elements. Using classical astronomic bols;

a = major se

e = eccentrici

i = inclination

Ω = longitude of (in the rence plane)

ω = argument of pe

M_o = mean anomaly at

f = true anomaly

We find the x components of position velocity

$$x = x_\omega P_x + y_\omega Q_x \tag{2.1}$$

$$\dot{x} = \dot{x}_\omega P_x + \dot{y}_\omega Q_x \tag{2.2}$$

with corresponding expressions for y z. The P and

P_x = $\cos\Omega \cos\omega$ - si $\sin\omega \cos i$

P_y = $\sin\Omega \cos\omega + \cos\Omega \sin\omega \cos i$

P_z = $\sin\omega \sin i$

Q_x = $-\cos\Omega \sin\omega - \sin\Omega \cos\omega \cos i$

Q_y = $-\sin\Omega \sin\omega + \cos\Omega \cos\omega \cos i$

Q_z = $\cos\omega \sin i$

and

$$x_\omega = r \cos f$$

$$y_\omega = r \sin f$$

$$\dot{x}_\omega = -\sqrt{\frac{\mu}{p}} \sin f$$

$$\dot{y}_\omega = \sqrt{\frac{\mu}{p}} (e + \cos f)$$

$$r = \frac{p}{1 + e \cos f}$$

$$p = a (1 - e^2)$$

The expansion of the true anomaly, f, in terms of the mean anomaly is given by:

$$f = M + e \left[2 + e^2 \left(3 + \frac{15}{4} e^2\right)\right] \sin M - e^3 \left[\frac{13}{3} + \frac{121}{6} e^2\right] \sin^3 M + \frac{e^2}{4} \left[5 + 27 \left(\frac{e}{4}\right)^2\right] \sin 2M - \frac{206}{3} \left(\frac{e}{4}^2\right)^2 \sin^2 M \sin 2 M + \frac{1097}{60} e^5 \sin^5 M \tag{2.3}$$

The mean anomaly at epoch is:

$$M_o = M_{1950.0} + n (JD_o - JD_{1950}) \tag{2.4}$$

$M_{1950.0}$ is computed from values of L and $\overline{\omega}$ found in the American Ephemeris, and M at any later time is

$$M (t) = M_o + n (t - t_o) \tag{2.5}$$

The mean elements of the orbits, referred to epoch of 1900, are expressible in the form

$$e (t') = e (1900) + \dot{e} t' + \dot{e}^2 \frac{(t')^2}{2} \tag{2.6}$$

$$t' = t - 1900.0$$

Table 2 gives a list of the coefficients which are built into the program for the various planets. Should higher accuracy in the planetary positions be required, one can insert osculating elements at a particular epoch.

BODY	Ω (deg)	$\dot{\Omega}$(deg/century)	ω (deg)	$\dot{\omega}$(deg/century)	M_o(1950) (deg)	e	i (deg)	n(rad/day)	Radius(er)*
MERCURY	47°.14528	1°.18500	28°.75305	0°.36944	-43°.50722	0.205615	7°.17667	.071425	.3653
VENUS	75.78361	0.90555	54.36861	0.50139	-49.28375	0.006818	3.39364	.027962	.9564
EARTH			101.21972	0.32057	- 2.48972	0.016750		.017202	1.0000
MARS	48.78667	0.77389	285.43167	1.06667	-190.80333	0.093310	1.85030	.0091461	.5354
JUPITER	99.44333	1.01055	273.27750	0.60000	302.63361	0.048335	1.30872	.0014502	10.818
SATURN	112.79028	0.87305	338.30805	1.08528	66.22611	0.055892	2.49253	.58399 x [illegible]$^{-3}$	[illegible].9797
URANUS	73.47722	0.49861	95.57278	1.11250	-71.546	0.0470	0.77247	.2047[illegible]$^{-3}$	3.8961
NEPTUNE	130.68139	1.09889	273.15161	-0.43222	150.786	[illegible]	1.77925	.10[illegible]$^{-3}$	3.[illegible]
PLUTO	108.95472	1.35805	113.84528	0.03084	-57.892	[illegible].247	17.14555	.06927 x [illegible]	[illegible]
MOON	259.18328	-1934.14201	75.14628	6003.17604	208.9967	0.054900	5.15000	.228027167	.2725

*Earth Radius
EPOCH: 1900.0 (except M_o)

**Values are quoted from Reference 3

Table 2

SOLAR SYSTEM CONSTANTS **

APPENDIX 3 DERIVATIVES OF OBSERVATIONS

This appendix lists the partial derivatives of the observations with respect to the position vector at the time of the observation. As pointed out earlier, the coefficients, $\left(\frac{\partial R_i}{\partial x_o}\right)$, are required for the least squares normal matrix. They are obtained by use of the chain rule for differentiation,

$$\left(\frac{\partial R_i}{\partial x_o}\right) = \left(\frac{\partial R_i}{\partial x}\right)\left(\frac{\partial x}{\partial x_o}\right)$$

The matrix $\left(\frac{\partial x}{\partial x_o}\right)$ was given in Appendix 1. $\left(\frac{\partial R_i}{\partial x}\right)$ for the various types of observables will be tabulated in the following.

Notation:

r - (x, y, z) - vector position of spacecraft with respect to the primary attracting center at time, t.

r_s - (x_s, y_s, z_s) - vector position of the radar site with respect to the center of the earth.

ρ - (ρ_x, ρ_y, ρ_z) - position of the spacecraft relative to the radar station

$\rho = r - r_s$ in earth phase

$= r - e - r_s$ in solar (lunar) phase

$= r + p - e - r_s$ in planet phases

σ - $(\sigma_x, \sigma_y, \sigma_z)$ - velocity of the spacecraft relative to the radar station

e - position of earth with respect to sun (moon)

p - position of planet with respect to sun

d_r - radius of curvature of the reference ellipsoid $d_r = d_r(\phi)$

S - sidereal time of station

λ - station longitud

ϕ - station geodeti tude

h - station altitude e the ref ellipsoid

a_e - equatorial radiu he ea

f - flattening of ea $\frac{1}{298.}$

ω_e - angular rotation e of t h

The radar station position ve is found

$$d_r = a_e \quad 1 - \left[(2f - f^2) \quad \phi\right]^{-1}$$

$$x_s = (d_r + h) \cos\phi \text{ co} \qquad - \omega_e y_s$$

$$y_s = (d_r + h) \cos\phi \text{ si} \qquad = \omega_e x_s$$

$$z_s = \left[(1-f)^2 d_r + h\right] \qquad = 0$$

The velocity components are not red one of the data types is doppler.

I Derivatives of the Observab th Res to r(t)

The radar observables are d by

$$\delta = \sin^{-1} \frac{\rho_z}{\rho} \qquad \text{loc clination}$$

$$\alpha = \tan^{-1} \frac{\rho_y}{\rho_x} \qquad \text{local right ascension}$$

$$H = S - \alpha \qquad \text{local hour angle}$$

$$E = \sin^{-1} \frac{\rho_r}{\rho} \qquad \text{elevation angle}$$

where $\rho_r = \frac{\rho' r_s}{|r_s|}$

$$A = \tan^{-1} \frac{(x_s \rho_y - y_s \rho_x)}{r_s \rho_z - z_s \rho_r} \qquad \text{azimuth from north}$$

$$\Delta f = \frac{k\rho'\sigma}{|\rho|}$$ doppler shift

$$|\rho| = \left[\rho'\rho\right]^{1/2}$$ slant range

By straightforward differentiation he a ve definitions, we find $\left(\frac{\partial R}{\partial x}\right)$

$$\frac{\partial \delta}{\partial x} = -\frac{\rho_x \rho_z}{|\rho|^2 u}$$

$$u = \left(\rho_x^2 + \rho_y^2\right)^{1/2}$$

$$\frac{\partial \delta}{\partial y} = -\frac{\rho_y \rho_z}{|\rho|^2 u}$$

$$\frac{\partial \delta}{\partial z} = \frac{u}{|\rho|^2}$$

$$\frac{\partial H}{\partial x} = \frac{\rho_y}{u^2}$$

$$\frac{\partial H}{\partial y} = -\frac{\rho_x}{u^2}$$

$$v = \left[|\rho|^2 - \rho_r^2\right]^{1/2}$$

$$\frac{\partial E}{\partial x} = \frac{1}{v}\left[\frac{x_s}{|r_s|} - \frac{\rho_x \rho_r}{|\rho|^2}\right]$$

$$\frac{\partial E}{\partial y} = \frac{1}{v}\left[\frac{y_s}{|r_s|} - \frac{\rho_y \rho_r}{|\rho|^2}\right]$$

$$\frac{\partial E}{\partial z} = \frac{1}{v}\left[\frac{z_s}{|r_s|} - \frac{\rho_z \quad \rho r}{|\rho|^2}\right]$$

$$\frac{\partial A}{\partial x} = \frac{1}{|r_s| v^2} \quad (z_s \rho_y - y_s \rho_z)$$

$$\frac{\partial A}{\partial y} = \frac{1}{|r_s| v^2} \quad (x_s \rho_z - z_s \rho_x)$$

$$\frac{\partial A}{\partial z} = \frac{1}{|r_s| v^2} \quad (y_s \rho_x - x_s \rho_y)$$

$$\frac{\partial \Delta f}{\partial x} = \frac{k}{|\rho|} \left(\sigma_x - \frac{\sigma_\rho}{|\rho|} \rho_x\right) , \qquad \frac{\partial \Delta f}{\partial \dot{x}} = \frac{k \rho_x}{\rho}$$

where $\sigma_\rho = \dfrac{\sigma' \rho}{|\rho|}$,

$$\frac{\partial \Delta f}{\partial y} = \frac{k}{|\rho|} \left(\sigma_y - \frac{\sigma_\rho}{|\rho|} \rho_y\right) , \qquad \frac{\partial \Delta f}{\partial \dot{x}} = \frac{k \rho_y}{\rho}$$

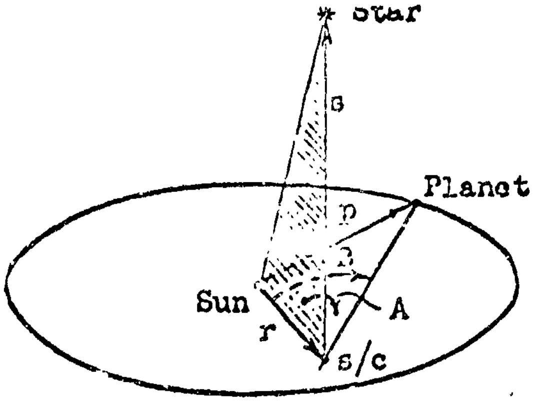
$$\frac{\partial \Delta f}{\partial z} = \frac{k}{|\rho|} \left(\sigma_z - \frac{\sigma_\rho}{|\rho|} \rho_z\right) , \qquad \frac{\partial \Delta f}{\partial \dot{z}} = \frac{k \rho_z}{\rho}$$

$$\frac{\partial \rho}{\partial x} = \frac{\rho_x}{\rho} , \qquad \frac{\partial \rho}{\partial y} = \frac{\rho_y}{\rho} , \qquad \frac{\partial \rho}{\partial z} = \frac{\rho_z}{\rho}$$

The above expressions are valid for observations with respect to a lunar station provided r_s and $\dot{v}_s$ are computed for the location of the station on the moon.

Spacecraft Angles

The clock angle, A, is defined by



$$\tan A = \frac{n_2' \, n_3}{n_2' \, n_1}$$

$$n_1 = \frac{s \times r}{|s \times r|} \qquad n_2 = \frac{p \times r}{|p \times r|} \qquad n_3 = \frac{n_1 \times r}{|r|}$$

s is the direction to a given star from the vehicle.

r is the position vector of the space craft with respect to the sun in this case.

The cone angle, B, is given by

$$-\cos B = \frac{r' \, (p - r)}{|r| \; |p - r|}$$

For the derivatives, we have:

$$\frac{\partial A}{\partial x} = \frac{1}{D} \left\{ |r| \; \left[|r|^2 \, (s'p) - (r'p) \, (r's) \right] \; (p \times s)_x \right.$$

$$- \; (p \times s'r) \left[(s'p) \; |r| x \; + \; (r'p) \; (r's) \frac{x}{|r|} - (r's) \; |r| p_x - (r'p) \; |r| s_x \right.$$

$$D = \left[|r|^2 - (r's)^2\right] \left[|r|^2 |p|^2 - (r'\right.$$

$$\frac{\partial B}{\partial x} = \frac{-\cos B}{\sqrt{1-\cos^2 B}} \left[\frac{x}{|r|^2} + \frac{2x - p_x}{r' (r - p} \quad \frac{x - p_x}{|p - r|^2}\right]$$

$\frac{\partial A}{\partial y}, \frac{\partial A}{\partial z}, \frac{\partial B}{\partial y}, \frac{\partial B}{\partial z}$ are obtained by replacing x by y, z in the corresponding expressions above.

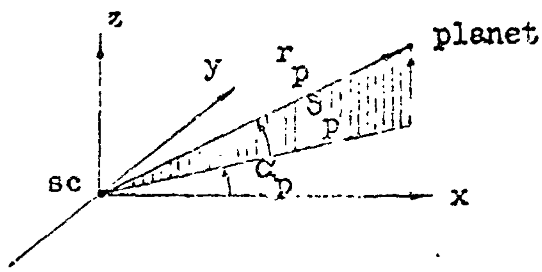
(Note that for the differential correction, the vectors n_1, n_2, n_3 need not be computed).

Occultations

An occultation of a planet is given by the two angles

$$\tan \alpha_p = \frac{-y_p}{-x_p}$$

$$\sin \delta_p = \frac{-z_p}{|r_p|}$$

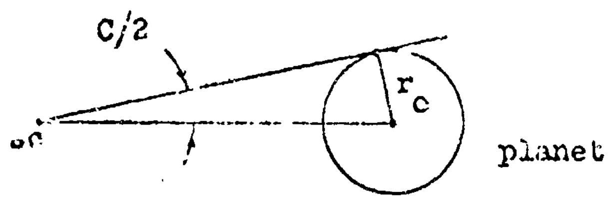


where r_p is the position of the spacecraft with respect to the planet being observed. The derivatives are the same as the ones given previously for α, δ but with replaced by $-r_p$.

Planetary diameters

The planetary diameter C is

$$\sin \frac{C}{2} = \frac{r_o}{|r_p|}$$



and

$$\frac{\partial C}{\partial x} = \frac{-2r_o\, x_p}{|r_I|^2 \left[|r_p|^2 - r_o^2\right]^{1/2}}$$

with corresponding expressions for y, z. Here r_o is the physical radius of the planet.

Derivatives of Observables with respect to [illegible]cal Constants

Station Coordinates on Earth:

$$\left(\frac{\partial R}{\partial \lambda}\right) = \left[\left(\frac{\partial R}{\partial x_s}\right) - \left(\frac{\partial R}{\partial \rho_x}\right)\right] \left(\frac{\partial x_s}{\partial \lambda}\right)$$

where λ denotes the vector (λ, ϕ, h) in [illegible] above matrices.

The components of $\left(\frac{\partial x_s}{\partial \lambda}\right)$ are

$$\frac{\partial x_s}{\partial \lambda} = -y_s \qquad \frac{\partial y_s}{\partial \lambda} = x_s \qquad \frac{\partial z_s}{\partial \lambda} = 0$$

$$\frac{\partial x_s}{\partial \phi} = -(d_r + h) \sin \phi \cos S$$

$$\frac{\partial y_s}{\partial \phi} = -(d_r + h) \sin \phi \sin S$$

$$\frac{\partial z_s}{\partial \phi} = \left[(1 - f)^2 \; d_r + h\right] \cos \phi$$

$$\frac{\partial x_s}{\partial h} = \cos \phi \cos S$$

$$\frac{\partial y_s}{\partial h} = \cos \phi \sin S$$

$$\frac{\partial z_s}{\partial h} = \sin \phi$$

$$\frac{\partial \dot{x}_s}{\partial \lambda} = -\omega_e \frac{\partial y_s}{\partial \lambda} \qquad \frac{\partial \dot{y}_s}{\partial \lambda} = \omega_e \frac{\partial x_s}{\partial \lambda}$$

$$\frac{\partial \dot{x}_s}{\partial \phi} = -\omega_e \frac{\partial y_s}{\partial \phi} \qquad \frac{\partial \dot{y}_s}{\partial \phi} = \omega_e \frac{\partial x_s}{\partial \phi}$$

$$\frac{\partial \dot{x}_s}{\partial h} = -\omega_e \frac{\partial y_s}{\partial h} . \qquad \frac{\partial \dot{y}_s}{\partial h} = \omega_e \frac{\partial x_s}{\partial h}$$

For the observational types $R = \alpha,\ \delta,\ H,\ \Delta f,$ ($\left(\frac{\partial R}{\partial x_s}\right) = 0;\ \left(\frac{\partial R}{\partial \rho_x}\right) = \left(\frac{\partial R}{\partial x}\right)$;

The latter were given previously. For the ty[illegible] and A, [illegible] given below:

$$\frac{\partial E}{\partial x_s} = \frac{1}{|r_s|\, v}\left[\rho_x - \ldots\ r\right]$$

$\frac{\partial E}{\partial y_s}$ and $\frac{\partial E}{\partial z_s}$ are obtained by replacing x by y and z respectively in the above.

$$\frac{\partial A}{\partial x_s} = \frac{1}{\left(x_s^{\,2} + y_s^{\,2}\right) v^2}\left[\rho_y \left(|r_s|\, \rho_z - z_s\, \rho_r\right) - \frac{\left(x_s\, \rho_y - y_s\, \rho_x\right)}{|r_s|}\left(x_s\, \rho_z - z_s\, \rho_x + \frac{x_s\, z_s}{|r_s|}\, \rho_r\right)\right]$$

$$\frac{\partial A}{\partial y_s} = \frac{-1}{\left(x_s^{\,2} + y_s^{\,2}\right) v^2}\left[\rho_x \left(r_s\, \rho_z - z_s\, \rho_r\right) \quad \frac{\left(x_s\, \rho_y - y_s\, \rho_x\right)}{|r_s|}\left(y_s\, \rho_z - z_s\, \rho_y + \frac{y_s\, z_s}{|r_s|}\, \rho_r\right)\right]$$

$$\frac{\partial A}{\partial z_s} = \frac{\rho_r}{v^2\, |r_s|^2}\left[x_s\, \rho_y - y_s\, \rho_x\right]$$

Determination of Physical tants

In the determination hysical constan are must be excercised in deciding which observation i which variab re being considered. For example, for the mass of t rth, μ_e, ight perform experiments which measure the g field on t rth and fro t decide how errors in measurements of the equatoria ius affect final estimation of error in the mass. Another way woul to conc consider a surface circular satellite and measure its asurement well as measuring the equatorial radiu inty in the mass. In the former case,

$$g = \frac{\quad}{a_e^{2}} .$$

$$\frac{\Delta g}{g} = 0 = \frac{\Delta\mu}{\mu} - \frac{2\Delta a_e}{a_e}$$

In the latter case,

$$P^2 \mu = 4\pi^2 a_e^3$$

$$\frac{2\Delta P}{P} = 0 = \frac{3\Delta a_e}{a_e} - \frac{\Delta\mu}{\mu} .$$

With these simple examples, we may see how differing measurements will yield differing relations between estimates of the physical constants.

For the case of the moon, we have at least four differing ways of estimating the mass of the moon. These are: (1) lunar equation, (2) parallactic inequality, (3) nutational effects upon the earth by the moon, and (4) period of the moon. The partial derivatives will differ according to which measurements we are making. Of the four estimations, we will adopt here the period-mass relationship.

$$P^2 \left(\mu_m + \mu_e\right) = 4\pi^2$$

$$\frac{\Delta\mu_m}{\mu_m} = \left(3\,\frac{\Delta a}{a} - \frac{\Delta\mu_e}{\mu_m + \mu_e}\right)\left(\quad\frac{+\mu_e}{\mu_e}\right) .$$

The uncertainty in the moon's mass is coup[illegible] [illegible]ether with the u[illegible]rtainty in the mass of the earth and the mean dist[illegible] [illegible]etween the earth and moon. The general philosophy here is that no estim[illegible]on be made of physical constants of another phase so that by this convention $\Delta\mu_e$ is set to zero.

Mass of the Sun

Let μ_s be the mass of the sun

μ_e be the mass of the earth

x_o be the initial position vector of the spacecraft in the sun frame

x_{bo} be the initial position vector of the spacecraft at burnout

e be the position vector of the earth in the sun frame

λ be the non-dynamical variables

au be the astronomical unit

From the definition of ρ, we have

$$\rho = r - e - r_s$$

$$\Delta\rho = \Delta r - \Delta e - \Delta r_s$$

Taking note of the fact that

$$r = (x_o, \mu_s) \text{ in the sun frame only}$$

$$x_o = x_o\,(x_{bo}, \mu_s, \mu_e)$$

$$e = e\,(\mu_s)$$

$$r_s = r_s\,(\lambda, \mu_e)$$

$$R = R\,(\rho)$$

so that

$$\Delta r = \left(\frac{\partial r}{\partial x_o}\right) \Delta x_o + \left(\frac{\partial r}{\partial \mu_s}\right)$$

$$\Delta x_o = \left(\frac{\partial x_o}{\partial x_{bo}}\right) \Delta x_{bo} + \left(\frac{\partial x_o}{\partial \mu_s}\right) \Delta \quad \left(\frac{\partial x_o}{\partial \mu}\right)$$

$$\Delta e = \left(\frac{\partial e}{\partial \mu_s}\right) \Delta \mu_s$$

$$\Delta r_s = \left(\frac{\partial r_s}{\partial \lambda}\right) \Delta \lambda + \left(\frac{\partial r_s}{\partial \mu_e}\right) \Delta \mu_e$$

$$\Delta R = \left(\frac{\partial R}{\partial \rho}\right) \Delta \rho$$

$$\Delta R = \left(\frac{\partial R}{\partial \rho}\right) \left\{ \left(\frac{\partial x_o}{\partial x_{bo}}\right) \Delta x_{bo} + \left[\left(\frac{\partial r}{\partial x_o}\right)\left(\frac{\partial x_o}{\partial \mu_s}\right) + \left(\frac{\partial r}{\partial \mu_s}\right) - \left(\frac{\partial r_s}{\partial \mu_s}\right)\right] \Delta \mu_s \right.$$

$$\left. + \left[\left(\frac{\partial r}{\partial x_o}\right)\left(\frac{\partial x_o}{\partial \mu_e}\right) - \left(\frac{\partial r_s}{\partial \mu_e}\right)\right] \Delta \mu_e - \left(\frac{\partial r_s}{\partial \lambda}\right) \Delta \lambda \right\}$$

This [illegible] is [illegible] times called the equations of conditions.

The coefficient of $\Delta \mu_s$ is:

$$\left[\left(\frac{\partial r}{\partial x_o}\right)\left(\frac{\partial x_o}{\partial \mu_s}\right) - \left(\frac{\partial e}{\partial \mu_e}\right) + \left(\frac{\partial r}{\partial \mu_s}\right)\right]$$

where

$$\left(\frac{\partial x_o}{\partial \mu_s}\right) = \left(\frac{\partial e}{\partial \mu_s}\right)_o = \left(\frac{\partial e}{\partial au}\right)_o \frac{au}{3\mu_s} = \frac{1}{3\mu_s} \quad (e)_o$$

The subscript "o" refers to the pos and velc of the earth at time of phase shift.

$$\frac{\partial e}{\partial \mu_s} = \frac{1}{3\mu_s} (e)_t$$

The subscript "t" refers to the positio velocity he earth at time of observation, t.

$$\mu_s \left(\frac{\partial R}{\partial \mu_s}\right) = \frac{\partial R}{\partial \rho} \left[\frac{1}{3} \left\{ \left(\frac{\partial r}{\partial x_o}\right) (e)_o - \right\} + \mu_s \left(\right. \right]$$

We see that uncertainties in the au aff he observat three ways:

1. $\frac{1}{3} \left(\frac{\partial r}{\partial x_o}\right) (e)_o$ is an initial or of injecti to the s n hase. This effect is sim to burnout er t the e .

2. (is the direct deviatio a nominal d n error e mass of the sun.

3. (is the error incurred w oordinates i he sun frame ar nsformed into coordin t the surface of the earth. In sun frame, errors in avitational field of the ear e neglected, (i.e., = 0).

Mass and Equatorial radius of the Earth

In solving for the mass of the earth, we require that

$$\frac{\partial a_e}{\partial \mu_e} = \frac{1}{2} \frac{a_e}{\mu_e} .$$

For any observation R,

$$\frac{\partial R}{\partial \mu_e} = \left(\frac{\partial R}{\partial x}\right)\left(\frac{\partial x}{\partial \mu_e}\right) + \left(\frac{\partial R}{\partial x_s}\right)\left(\frac{\partial x_s}{\partial a_e}\right) \frac{\partial a_e}{\partial \mu_e} .$$

For our purposes, it is sufficiently accurate to assume

$$\left(\frac{\partial x_s}{\partial a_e}\right) = \left(\frac{x_s}{a_e}\right)$$

hence on combining,

$$\mu_e\left[\frac{\partial R}{\partial \mu_e}\right] = \mu_e \left(\frac{\partial R}{\partial x}\right)\left(\frac{\partial x}{\partial \mu_e}\right) + \left(\frac{\partial R}{\partial x_s}\right)\left(\frac{x_s}{2}\right)$$

Mass of the Moon

$$\mu_m\left[\frac{\partial R}{\partial \mu_m}\right] = \mu_m \frac{\partial R}{\partial \rho} \left(\frac{\partial \rho}{\partial \mu_m}\right)$$

This case is analogous to the case of the solar mass except that μ_s is replaced by the sum of the earth-moon mass. Hence, if e is the position o· the earth relative to the moon, then

$$\mu_m \left(\frac{\partial R}{\partial \mu_m}\right) = \left(\frac{\partial R}{\partial \rho}\right) \left\{ \frac{1}{3\left(\frac{\mu_e}{\mu_m} + 1\right)} \left[\left(\quad (e_o) - (e_t) \right] + \mu_m \left(\frac{\partial x}{\partial \mu_m}\right) \right\}$$

Strictly speaking, the uncertainty in the h mass as well as mass of the significantly perturbs the mean distance moon. These effects are ignored, so that we have only accounted for the r y in the earth-moon distance due to uncertainties in the moon's mass y. is all we can do because of the rule that only uncertainties in prim ass center will be considered in any one phase.

Velocity of Light c_o

c_o enter in the doppler formula

$$\Delta f = - k \sigma_\rho$$

hence,

$$c_o \frac{\partial \Delta f}{\partial c_o} \doteq k \sigma_\rho$$

and also in the slant range through time del τ.

$$\frac{c_o}{2} \tau = |\rho|$$

$$c_o \frac{\partial \tau}{\partial c_o} = - k' |\rho|$$

If k and k' are set to -1, the observables are change τ to σ_ρ and $|\rho|$.

APPENDIX 4 THE IMPACT VECTOR AND ITS SENSITIVITY COEFFICIENTS

Computation of the Impact Vector (b)

(The exact definition of b is given in (4.4) below)

Let $r = (x, y, z)$ and $v = (\dot{x}, \dot{y}, \dot{z})$ be the position and velocity respectively of the spacecraft with respect to the target planet at the point where b is to be computed. This point should be sufficiently removed from the target planet such that v is a good approximation to v_∞, the velocity at infinity of the spacecraft with respect to the planet on the incoming hyperbola. In the program, b is computed at the initial time point in the target centered phase. $|r|$ at this instant is equal to or greater than the radius of the sphere of action of the target.

The impact parameter, B, in the reference cartesian system is approximately*

$$B = r - \left(\frac{r'v}{|v|^2}\right) v \tag{4.1}$$

(A prime denotes transpose and $r'v$ is the inner product of r and v; $r'r = |r|^2$.) It is frequently convenient to express B in terms of impact parameter (B) plane coordinates. The specification of such a system requires a second reference plane which intersects the B plane. One possible choice is to choose the ecliptic as the second reference plane. This choice is used here and the B plane system is illustrated in Figure 2.

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*The exact definition of the B vector is given by

$$J = B \times v_\infty = r \times v$$

where J is the angular momentum. Hence cross multiplying by v_∞ and expanding, one obt[illegible]ns

$$v_\infty^2 B = v_\infty \times (r \times v) = (v_\infty' v) \quad r - (v_\infty' r) v$$

$$B = \left(\frac{v_\infty' v}{|v_\infty|^2}\right) r - \left(\frac{v_\infty' r}{|v_\infty|^2}\right) v$$

If the particle is at a sufficient distance such that $v \simeq v_\infty$, then one obtains the expression for B in (4.1) above.

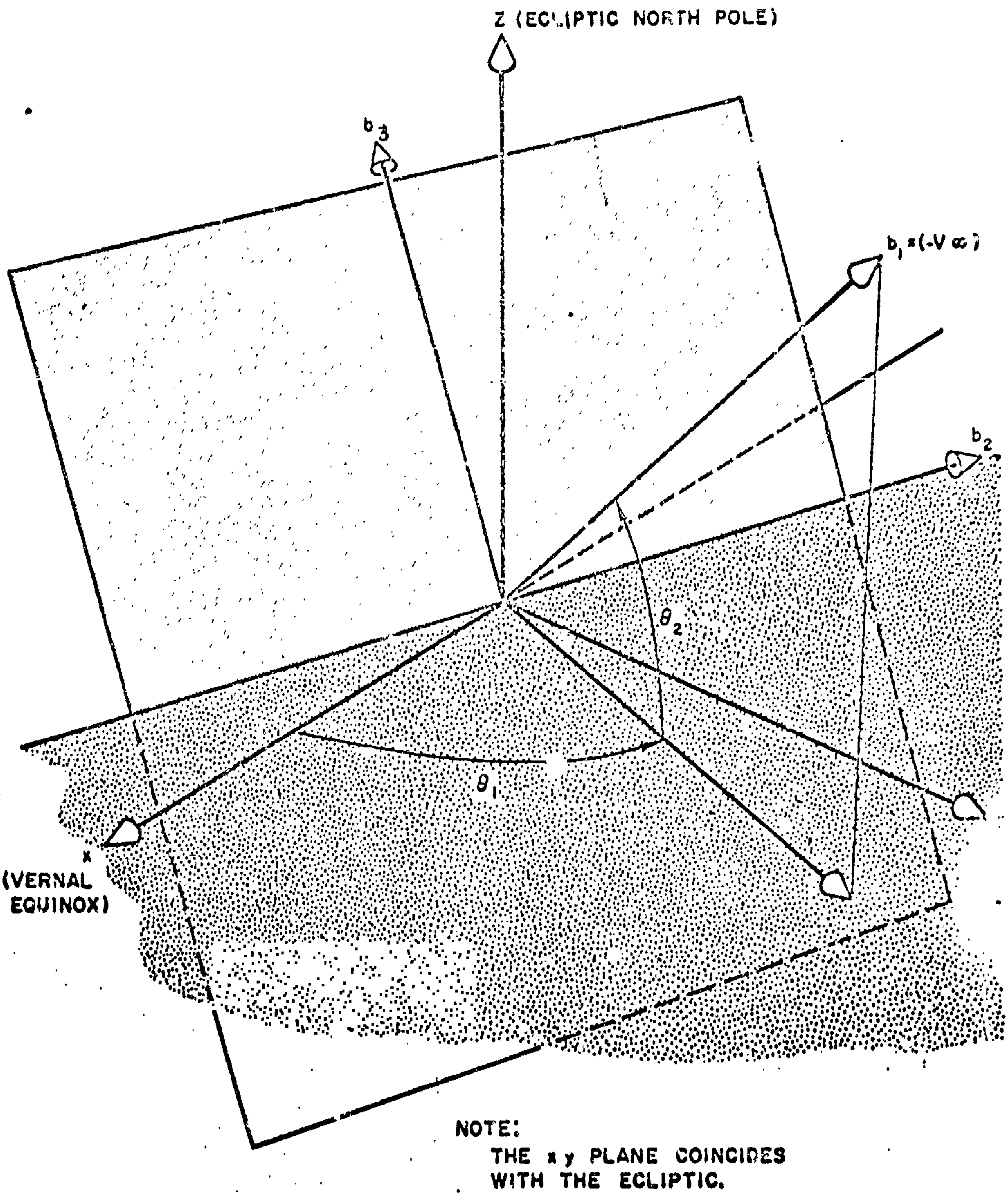

FIGURE 2. Illustration of the B-Plane Coordinate System

The transformation to the B plane system is given by three successive rotations

$$\tilde{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \gamma\,\beta\,\alpha\,B \tag{4.2}$$

where

$$\gamma\,\beta\,\alpha = \underset{\theta_2}{\begin{pmatrix} C & 0 & S \\ 0 & 1 & 0 \\ -S & 0 & C \end{pmatrix}} \underset{\theta_1}{\begin{pmatrix} C & S & 0 \\ -S & C & 0 \\ 0 & 0 & 1 \end{pmatrix}} \underset{\epsilon}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & C & S \\ 0 & -S & C \end{pmatrix}} \tag{4.3}$$

in which $C = \cos$; $S = \sin$; and the arguments are given beneath the rotation matrices; they are defined by

$$\epsilon = \text{obliquity of ecliptic} \sim 23.5°$$

$$\theta_1 = \tan^{-1}\left(\frac{-\dot{y}}{-\dot{x}}\right)$$

recall that $v = v_\infty$

$$\theta_2 = \sin^{-1}\left(\frac{-\dot{z}}{v}\right)$$

α is just a rotation from the original reference plane (earth's equatorial) to the ecliptic. β and γ are rotations through the two polar angles of $-v$ as shown in the figure. In the new system b_1 is along $-v$ and is zero or very small; b_2 is along the intersection of the B plane and the ecliptic, and b_3 completes a right handed cartesian system.

We shall define a new vector b with elements

$$b = \begin{pmatrix} m_1 \\ m_2 \\ t_f \end{pmatrix} \qquad \begin{aligned} m_1 &= b_2 \\ m_2 &= b_3 \end{aligned} \tag{4.4}$$

which is called the impact vector. t_f is the time of flight from the initial epoch to the target point (or some appropriate point of interest.)

Partial Derivatives of b

To find the derivatives of b with respect to initial conditions, x_o, we again make use of the chain rule.

$$\left(\frac{\partial\tilde{b}}{\partial x_o}\right) = \left(\frac{\partial\tilde{b}}{\partial x}\right)\left(\frac{\partial x}{\partial x_o}\right) \tag{4.5}$$

where x is the combined vector, x = (r, v). Consider first the derivatives for b_2 and b_3. They are obtained by differentiation of (4.2) in which we consider the rotation matrices as constants. Hence

$$\left(\frac{\partial\tilde{b}}{\partial x_o}\right) = \gamma\,\beta\,\alpha\left(\frac{\partial B}{\partial x}\right)\left(\frac{\partial x}{\partial x_o}\right) \tag{4.6}$$

In components, $\left(\frac{\partial B}{\partial x}\right)$ is:

[x below refers specifically to the coordinate x and is not to be confused with the convenient abbreviation for the combined vector (r, v) used above.]

$$\frac{\partial B_x}{\partial \dot{x}} = 1 - \frac{\dot{x}^2}{|v|^2}$$

$$\frac{\partial B_x}{\partial \dot{y}} = - \frac{\dot{x}\,\dot{y}}{|v|^2}$$

$$\frac{\partial B_x}{\partial \dot{z}} = - \frac{\dot{x}\,\dot{z}}{|v|^2}$$

$$\frac{\partial B_x}{\partial x} = \frac{r'v}{|v|^2}\left[\frac{2\,\dot{x}^2}{|v|^2} - 1\right] - \frac{x\dot{x}}{|v|^2}$$

$$\frac{\partial B_x}{\partial \dot{y}} = \left(\frac{r'v}{|v|^4}\right) 2\dot{x}\,\dot{y} - \frac{\dot{x}\,y}{|v|^2}$$

$$\frac{\partial B_x}{\partial \dot{z}} = \left(\frac{r'v}{|v|^4}\right) 2\dot{x}\,\dot{z} - \frac{\dot{x}\,z}{|v|^2}$$

Expressions for the derivatives of B_y, B_z are obtained by an exchange of letters in the above. From (4.6), the derivatives for b_2 and b_3 follows. In (4.5) the matrix $\left(\frac{\partial x}{\partial x_0}\right)$ is just the variational matrix evaluated at the computation time. This was given in Appendix 1.

If μ is a mass constant for a phase preceding the target phase, the μ derivatives of b_2 and b_3 are obtained from .

$$\left(\frac{\partial b}{\partial \mu}\right) = \gamma\,\beta\,\alpha\left(\frac{\partial B}{\partial x}\right)\left(\frac{\partial x}{\partial \mu}\right) \qquad (4.7)$$

The only difference between (4.7) and (4.6) is the last factor. If μ_f is the mass constant of the target phase, then $\left(\frac{\partial b}{\partial \mu_f}\right) = 0$.

For the partials of t_f, we consider two separate cases. The first is one in which we find the changes in the flight time prior to the target phase due to incremental changes in orbit parameters preceding the target phase. The second case deals with changes of flight time in the target phase due to variations at the initial point of the target phase. The latter is also useful for finding the derivatives of flight time in the geocentric phase when the velocity vector at infinity is varied in the course of a search routine. (See Appendix 5).

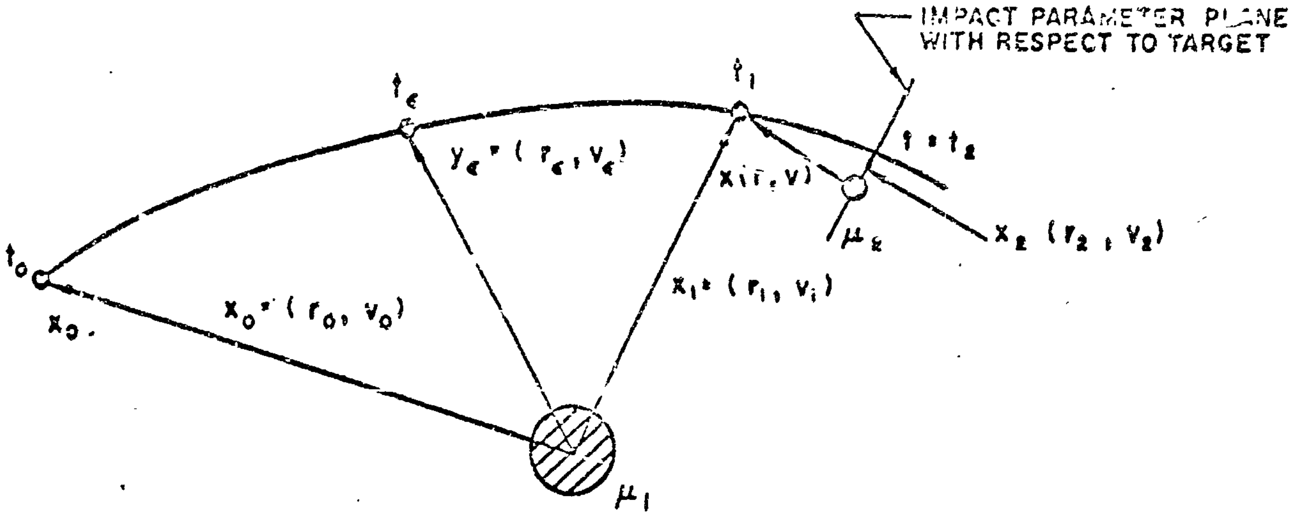

Figure 3. Geometry of the Transfer Orbit

Referring to Figure 3, we define

μ_1, μ_2	-	mass constants of the two bodies involved
$x_o = (r_o, v_o)$	-	the initial position and velocity of the spacecraft in the transfer orbit. $(t = t_o)$
$x = (r, v)$	-	spacecraft with respect to target planet at initial epoch of target phase $(t = t_1)$
$x_2 = (r_2, v_2)$	-	spacecraft with respect to target at the crossing of the B plane $(t = t_2)$

<u>Case 1</u>

In the transfer orbit, let $t_f = t_1 - t_o$. The partials of t_f are

$$\left(\frac{\partial t_f}{\partial x_o}\right) = \frac{-\ v' \left(\frac{\partial r}{\partial x_o}\right)}{|v|^2} \tag{4.8}$$

This is simply the variation of the position vector in the direction of the approach asymptote to the target divided by the hyperbolic excess velocity. (Again r is large enough so that v is a good approximation to the direction of the asymptote.) The negative sign occurs because a positive variation (in the same direction as v) implies that it requires less time to reach a reference distance along the orbit. Also we have

$$\left(\frac{\partial t_1}{\partial \mu_1}\right) = \frac{-v'\left(\frac{\partial r}{\partial \mu_1}\right)}{|v|^2}. \tag{4.9}$$

Case 2

In the target phase we let $t_g = t_2 - t_1$, then for an independent change δr

$$\left(\frac{\partial t_g}{\partial r}\right) = \frac{-v'}{|v|^2} \tag{4.10}$$

$$\left(\frac{\partial t_g}{\partial v}\right) = \frac{-v'\left(\frac{\partial r}{\partial v_2}\right)\left(\frac{\partial v_2}{\partial v}\right)}{|v|^2} = \frac{-v'\left(\frac{\partial r}{\partial v_2}\right)\left(\frac{\partial r}{\partial r_2}\right)'}{|v|^2} \tag{4.11}$$

The last equality is the result of a matrix inversion theorem given in Reference [4]. One way to arrive at (4.11) is by considering the symmetrical escape trajectory with initial point $(\tilde{r}_2, \tilde{v}_2)$ and final point $(\tilde{r}, \tilde{v})$. The time, $\tilde{t}_g$, to go from $(\tilde{r}_2, \tilde{v}_2)$ to $(\tilde{r}, \tilde{v})$ is the same as the time to go from (r, v) to (r_2, v_2).

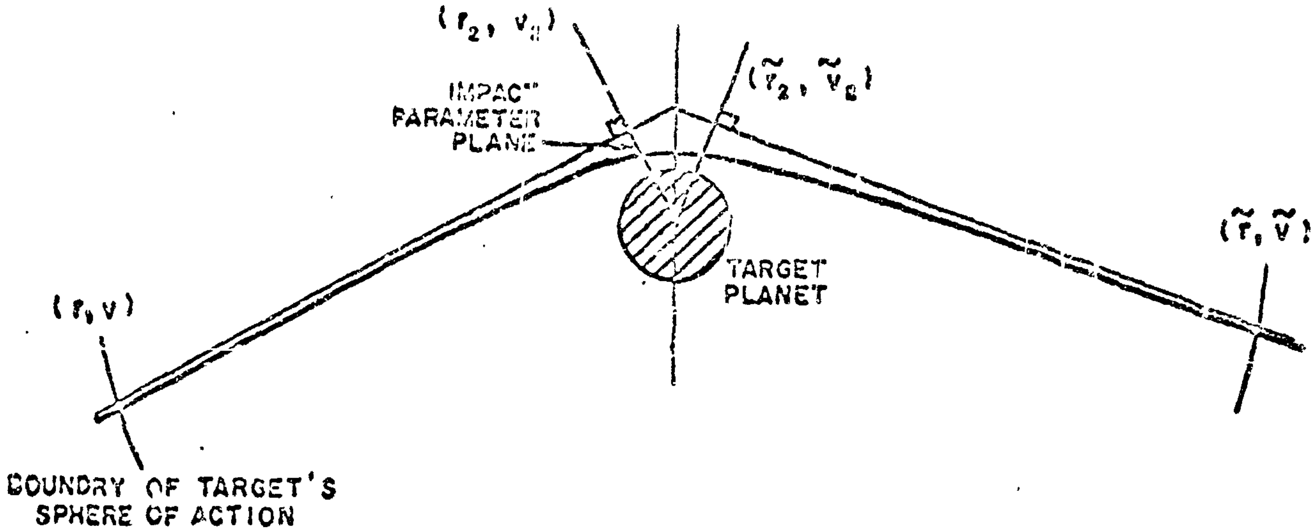

Figure 4. Geometry of Orbit in Target Phase

A variation δv in the approach trajectory produces a change δv_2 at time t_2 along the nominal. An increment $\delta \tilde{v}_2$ causes a change in time of flight $\delta \tilde{t}_g$ given by $\delta \tilde{t}_g = \left(\frac{\partial \tilde{t}_g}{\partial \tilde{v}_2}\right) \delta \tilde{v}_2$ where

$$\left(\frac{\partial \tilde{t}_g}{\partial \tilde{v}_2}\right) = \frac{-\tilde{v}' \left(\frac{\partial \tilde{r}}{\partial \tilde{v}_2}\right)}{|\tilde{v}|^2} = \frac{-v' \left(\frac{\partial r}{\partial v_2}\right)}{|v|^2} = \left(\frac{\partial t_g}{\partial v_2}\right) \tag{4.12}$$

Hence using the chain rule in (4.12) yields (4.11).

There are two μ derivatives for t_g. We have

$$\left(\frac{\partial t_g}{\partial \mu_1}\right) = \frac{-v' \left(\frac{\partial r}{\partial v_2}\right)\left(\frac{\partial v_2}{\partial \mu_1}\right)}{|v|^2} \tag{4.13}$$

And

$$\left(\frac{\partial t_g}{\partial \mu_2}\right) = \frac{-v'\left(\frac{\partial r}{\partial v_2}\right)\left(\frac{\partial v_2}{\partial \mu_2}\right)}{|v|^2} \qquad (4.14)$$

There are no changes in t_g as a result of positional displacements from μ_1 at t_1 because any displacement here produces a change $\delta\mu_1$ which in effect brings the spacecraft position at t_1 to its nominal starting point for the beginning of the target phase.

Within any given phase, for a change of the epoch from t_o to another epoch t_ϵ, the t_f derivatives with respect to new initial conditions, y_o, can be easily obtained by use of the chain rules. For example in Figure 4, if $t_f = t_1 - t_\epsilon$

$$\left(\frac{\partial t_f}{\partial y_o}\right) = \frac{-v'\left(\frac{\partial r}{\partial y_o}\right)}{v'v} = \frac{-v'\left(\frac{\partial r}{\partial x_c}\right)\left(\frac{\partial x_o}{\partial y_o}\right)}{v'v}$$

The μ derivatives are not related as simply and have to be evaluated at each epoch.

The Critical Plane and the Principal Directions for Midcourse Maneuvers

At a point (t, r, v) along a rendezvous trajectory, an impulse $\delta v = (\delta\dot{x}, \delta\dot{y}, \delta\dot{z})$ will cause a change in the b vector at the target given by

$$\delta\tilde{b} = \left(\frac{\partial \tilde{b}}{\partial \tilde{v}}\right) \delta v \qquad (4.15)$$

For definition of $\tilde{b}$, see (4.2).

$\left(\frac{\partial\tilde{b}}{\partial v}\right)$ may be written

$$\frac{\partial\tilde{b}}{\partial v} = \begin{pmatrix} \frac{\partial b_1}{\partial v} \\ \frac{\partial b_2}{\partial v} \\ \frac{\partial b_3}{\partial v} \end{pmatrix}$$

Again, $\left(\frac{\partial b_1}{\partial v}\right)$ should be zero or very small and may be neglected. The critical plane has the vector normal given by

$$n = \left(\frac{\partial b_2}{\partial v}\right) \times \left(\frac{\partial b_3}{\partial v}\right) \; ; \tag{4.16}$$

the component $\delta v_n = \frac{\delta v' n}{|n|}$ will cause no change in $\tilde{b}$. The magnitude of $\delta\tilde{b}$ in (4.15) is given by

$$\left[\delta\tilde{b}'\delta\tilde{b}\right]^{1/2} = \left[\delta v' \left(\frac{\partial\tilde{b}}{\partial v}\right)' \left(\frac{\partial\tilde{b}}{\partial v}\right) \delta v\right]^{1/2} \tag{4.17}$$

If $\left(\frac{\partial\tilde{b}}{\partial v}\right)' \left(\frac{\partial\tilde{b}}{\partial v}\right) = M$ is diagonalized by an orthogonal transformation to the form

$$\Lambda = U'MU = \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & 0 \end{pmatrix}$$

where $\lambda_1 > \lambda_2$, then the columns of the matrix U are the eigenvectors or the principal directions. The first eigenvector, $\hat{e}_1$ is in the direction of maximum sensitivity; $\hat{e}_2$ is in the direction of minimum sensitivity, both of which are in the critical plane. $\hat{e}_3$ is along n and is the direction of zero sensitivity for the miss components (b_2, b_3).

One of the program options is to compute these sensitivity coefficients and the principal directions at specified points along the orbit. To do this at a sequence of time points t_i, we need to find $\left(\frac{\partial b}{\partial v_i}\right)$. This is done by finding

$$\left(\frac{\partial b}{\partial v_i}\right) = \left(\frac{\partial b}{\partial x_o}\right)\left(\frac{\partial x_o}{\partial v_i}\right) .$$

$\left(\frac{\partial x_o}{\partial v_i}\right)$ is most easily obtained from $\left(\frac{\partial x_i}{\partial x_o}\right)^{-1} = \left(\frac{\partial x_o}{\partial x_i}\right)$

where

$$\left(\frac{\partial x_o}{\partial x_i}\right) = \begin{pmatrix} \dfrac{\partial x_o}{\partial r_i} \\ ----- \\ \dfrac{\partial x_o}{\partial v_i} \end{pmatrix} .$$

APPENDIX 5 SEARCH ROUTINE

The search routine is used to achieve a set of required final conditions by means of an iterative differential correction procedure. The search may be carried out by varying either the initial condition vector, x_o, or the velocity at infinity, v_∞, on the geocentric escape hyperbola. The former method may be used with any initial orbit phase, the latter is restricted to orbits leaving the earth.

Consider first the search on x_o. To start the search, it is necessary to input the required final vector, b_r, and a set of initial conditions which yield terminal conditions, b, reasonably close to b_r. The program then finds the linear correction

$$\delta x_o = \left(\frac{\partial b}{\partial x_o}\right)^{-1} \left[b_r - b\ (x_o)\right] = \left(\frac{\partial b}{\partial x_o}\right)^{-1} \delta b. \tag{5.1}$$

$x_{o1} = x_o + \delta x_o$ is used as the next trial value. This process is repeated until $\left|b\ (x_o) - b_r\right|$ is less than some preassigned quantity. In Appendix 4, b was defined as a 3 vector. x_o is a 6 vector. Hence, x_o is not determined uniquely by a specification of b_r alone. Also, the search may be done with or without fixing the time of flight. In the latter case, only two independent components of x_o need be varied. An option is provided whereby the components of x_o (2 or 3) may be designated for the search. If the components are not specified, then the program varies all six initial conditions and finds the correction which has the smallest total magnitude. This is done in the following way. Let Q be the (3 x 6) matrix $Q = \left(\frac{\partial b}{\partial x_o}\right)$; U an orthogonal transformation which defines new variables

$$y = U' x_o \tag{5.2}$$

such that

$$\Lambda = U'Q'QU \tag{5.3}$$

is a diagonal matrix. From equation (5.1)

$$\delta b'\delta b = \delta x_o' \, Q'Q \, \delta x_o = \delta y'\Lambda\delta y = \sum_{i=1}^{6} \lambda_i \, \delta y_i^2$$

where λ_i, δy_i are the components of Λ and δy. If we set the three components of y_i with the smallest eigenvalues equal to zero and solve for the three remaining components y_3 from

$$\delta y_3 = (QU)_3^{-1} \, \delta b \quad , \tag{5.4}$$

the resultant magnitude of the change $\delta y_3' \delta y_3$ will have the smallest possible value. δx_o can then be obtained from (5.2). The process is equally valid if only two components are varied (free time of flight).

The eigenvalues in (5.3) depend on the scaling of the independent variables. In the program (5.2) is actually given by

$$y = SU'x_o \tag{5.5}$$

where S is a diagonal matrix of scaling factors. In the program we also allow for a search in terms of the initial spherical coordinates defined in Appendix 1. The matrix S is especially important in this case since it allows for relative weights between angles and distances to be adjusted. In the cartesian system using megameters and kilometers/second as the units, the identity matrix seems to be a good S matrix.

While the search on x_o is usually satisfactory in terms of achieving the final conditions, the resultant x_o's may not always correspond to realistic launch conditions at the earth. For this reason a search on the velocity at infinity on the geocentric escape hyperbola may be substituted. Since this velocity vector does not determine the geocentric conic uniquely, the other orbital elements may be chosen so as to match a specified powered flight phase. If v_∞ is the velocity vector at infinity, the search routine finds the correction vector Δv_∞

$$\Delta v_\infty = \left(\frac{\partial b}{\partial v_\infty}\right)^{-1} \left\{ b_r - b\,(v_\infty) \right\}$$

and iterates for the correct value. After completing the search, the routine then calculates the injection conditions near the earth with certain specified constraints. Since v_∞ is a 3 vector; the solution for Δv_∞ in terms of b_r is unique if flight time is fixed. Otherwise, there are 3 choices for the combinations of the two components of v_∞ to be varied. The particular combination may be preselected or they can all be varied by the machine on the basis of a minimal magnitude criterion described earlier.

A fundamental block diagram of the search routine is shown in Figure 7. A somewhat more elaborate search routine which does not require an initial estimate, but only launch and flight time, is being contemplated. It has been postponed until other more urgent aspects of the program are completed since other existing analytic lunar and interplanetary programs are available to supply the initial condition estimates needed by TAPP.

Partial Derivatives for Search Routine

In the actual search on initial conditions, the independent variables may be the cartesian (x_o) or spherical (s_o) coordinates at injection. If cartesian, the matrix $\left(\frac{\partial b}{\partial x_o}\right)$ is given in Appendix 4. If spherical, the matrix $\left(\frac{\partial b}{\partial s_o}\right)$ is

$$\left(\frac{\partial b}{\partial \varepsilon_c}\right) = \left(\frac{\partial b}{\partial x_o}\right)\left(\frac{\partial x_o}{\partial \varepsilon_o}\right) \quad , \qquad (5.6)$$

where $\left(\frac{\partial x_o}{\partial \varepsilon_o}\right)$ is a 6 x 6 matrix given in Reference [5].

If the search is on v_∞, the derivatives for the two miss components $\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$ are given by

$$\frac{\partial}{\partial v_\infty} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \frac{\partial}{\partial v_1} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} . \qquad (5.7)$$

where v_1 is the velocity relative to the earth at the pseudo infinity point. This is the point at which the spacecraft leaves the earth's sphere of action for planetary trips. It is the initial point on the moon phase conic for lunar missions.

The time of flight derivatives are also given in Appendix 4 with one exception. We need to find the time of flight change in the earth phase due to an independent variation of δv_1. We first find the change δv_o at injection needed to produce the increment δv_1; then the variation δr_1 of the position vector at the pseudo infinity point due to δv_o and divide by v_1 to find the change in flight time.

$$\frac{\partial t_f}{\partial v_\infty} \cong \frac{\partial t_f}{\partial v_1} = \frac{-v_1'}{v_1 \cdot v_1} \left(\frac{\partial r_1}{\partial v_o}\right)\left(\frac{\partial v_o}{\partial v_1}\right) \qquad (5.8)$$

Equation (5.8) is the same as Equation 4.11 of Appendix 4 except for a change of notation.

If the motion relative to the target is elliptic instead of hyperbolic, the components (m_1, m_2) of the b vector are replaced by the target centered right ascension and declination. The partial derivatives which replace the ones for m_1 and m_2 are

$$\frac{\partial}{\partial x_o}\begin{pmatrix}\alpha\\ \delta\end{pmatrix} = \frac{\partial}{\partial r_f}\begin{pmatrix}\alpha\\ \delta\end{pmatrix}\left(\frac{\partial r_f}{\partial x_o}\right) ,$$

where r_f is the nominal position vector at the final impact point, $\left(\frac{\partial r_f}{\partial x_o}\right)$ is the (3 x 6) variational matrix evaluated at that point and

$$\frac{\partial}{\partial r_f}\begin{pmatrix}\alpha\\ \delta\end{pmatrix} = \begin{pmatrix}\frac{\partial\alpha}{\partial x_f} & \frac{\partial\alpha}{\partial y_f} & 0\\ \frac{\partial\delta}{\partial x_f} & \frac{\partial\delta}{\partial y_f} & \frac{\partial\delta}{\partial z_f}\end{pmatrix}$$

Expressions for these are given in Appendix 1, page 27 .

Derivation of Injection Conditions from the v_∞ Vector

After an acceptable v_∞ vector has been found in the search routine, it is necessary to derive the injection conditions(position and velocity at end of final burnout) which correspond to v_∞. (Actually, the initial v_∞ vector is specified by a magnitude and two angles. The search routine however varies the cartesian components.) The whole object of the computation is to find injection conditions which yield the correct v_∞ vector while at the same time have a prescribed launch azimuth, and launch latitude. This is accomplished by adjustment of the coast time in a circular parking orbit and the launch time. The computation procedure is essentially due to V. C. Clarke. [Reference 6]. The inputs are:

C_3 = twice the vis-viva energy

α_S = right ascension of asymptote

δ_S = declination of the asymptote

C_3, α_S, and δ_S specify the initial v_∞ vector. The remaining constants which specify the constraints are:

A_L = launch azimuth

ϕ_L = launch latitude

λ_L = launch longitude

Γ = injection path angle of velocity from local horizontal

R = geocentric distance at injection

t_{02} = time from launch to first burnout

f_{02} = true anomaly between launch and first burnout

t_{23} = time of second burn

f_{23} = angle swept out during second burn

R_I = Pseudo infinity distance = 380 Mm

n = mean angular motion of satellite in circular (100 naut. mile) parking orbit

ω_e = rotational rate of the earth

μ_e = gravitational constant of the earth.

Figure 5 illustrates the geometry of the parking and final orbit.

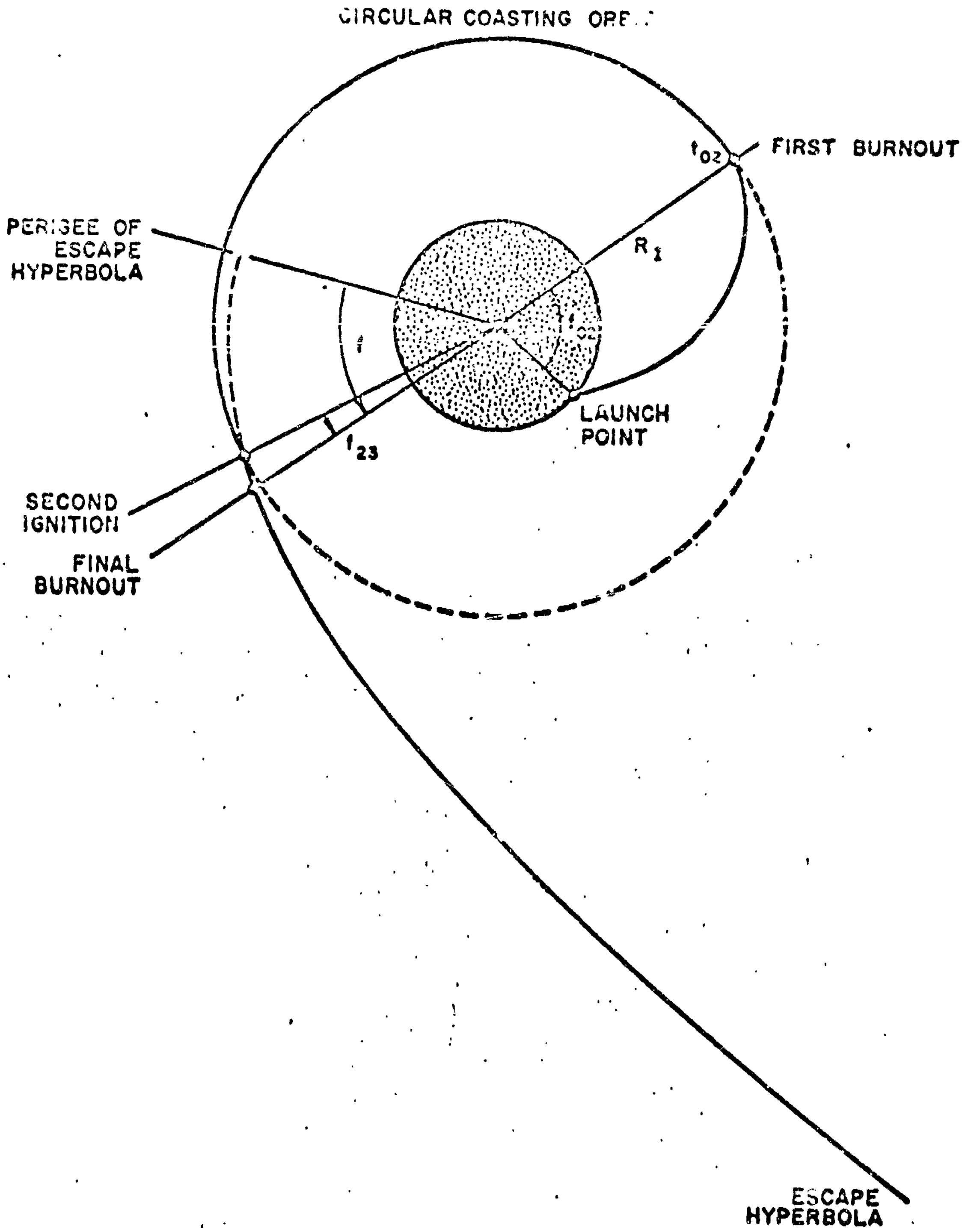

FIGURE 5. Geometry of the Parking and Escape Orbit

Equations to find final burnout conditions at the earth from C_3, α_S, δ_S follow:

The cartesian components of the asymptote direction $\bar{S}$ are

$$
\begin{aligned}
S_x &= \cos \delta_S \cos \alpha_S \\
S_y &= \cos \delta_S \sin \alpha_S \\
S_z &= \sin \delta_S
\end{aligned}
\qquad (5.9)
$$

The unit normal to the orbit plane is $\bar{W}$ where

$$W_z = \cos \phi_L \sin A_L = \cos i$$

$$\Omega = \alpha_s - \text{arc sin} \left[\frac{W_z}{+\sqrt{1 - W_z^2}} \; \frac{S_z}{\sqrt{1 - S_z^2}} \right] \quad (-90^\circ \leq (\alpha_s - \Omega) \leq +90^\circ)$$

$$
\begin{aligned}
W_y &= -\cos \Omega \sqrt{1 - W_z^2} \\
W_x &= \sin \Omega \sqrt{1 - W_z^2}
\end{aligned}
\qquad (5.10)
$$

Define the vector $\bar{B}$ by

$$\bar{B} = \bar{S} \times \bar{W} \qquad (5.11)$$

Compute

$$V = \left[C_3 + \frac{2\mu_e}{R} \right]^{1/2} \qquad (5.12)$$

$$p = \frac{V^2 R^2 \cos^2 \Gamma}{\mu_e} \qquad (5.13)$$

$$e^2 = 1 + \frac{p\, C_3}{\mu_e} \qquad (5.14)$$

For hyperbolic cases, the true anomaly of the asymptote is f_a where

$$\cos f_a = -\frac{1}{e}, \quad \sin f_a = \left[1 - \cos^2 f_a\right]^{1/2} . \tag{5.15}$$

For elliptic cases: (5.15) is replaced by

$$\cos f_a = \frac{p - R_I}{eR_I}; \quad \sin f_a = \left[1 - \cos^2 f_a\right]^{1/2} . \tag{5.16}$$

Next compute the vectors

$$\begin{aligned} \overline{P} &= \cos f_a \overline{S} + \sin f_a \overline{B} \\ \overline{Q} &= \sin f_a \overline{S} - \cos f_a \overline{B} \end{aligned} \tag{5.17}$$

The true anomaly at injection is f where

$$\cos f = \frac{p - R}{eR}, \qquad \sin f = \frac{V}{e} \sqrt{\frac{p}{\mu}} \sin \Gamma \tag{5.18}$$

Finally, the position and velocity at injection are:

$$\overline{R} = R \cos f \; \overline{P} + R \sin f \, \overline{Q} \tag{5.19}$$

$$\dot{\overline{R}} = -\sqrt{\frac{\mu}{p}} \sin f \, \overline{P} + \sqrt{\frac{\mu}{p}} (e + \cos f) \, \overline{Q} \tag{5.20}$$

The right ascension of the launch site is given by α_L where

$$\cos \alpha_L = \frac{W_x \sin \phi_L \sin A_L + W_y \cos A_L}{W_z^2 - 1} \tag{5.21}$$

$$\sin \alpha_L = \frac{W_y \sin \phi_L \sin A_2 - W_x \cos A_L}{W_z^2 - 1} , \qquad (5.22)$$

and the unit vector to the launch site is $\bar{R}_L$ with components

$$\begin{aligned} x_L &= \cos \alpha_L \cos \phi_L \\ y_L &= \sin \alpha_L \cos \phi_L \\ z_L &= \sin \phi_L \end{aligned} \qquad (5.23)$$

The true anomaly of the launch site in the orbit plane f_L is obtained from

$$f_L = \tan^{-1} \frac{\bar{R}_L \cdot \bar{Q}}{\bar{R}_L \cdot \bar{P}} . \qquad (5.24)$$

Define Δf by

$$\Delta f = 2\pi - f_L + f .$$

The coast time is given by

$$t_c = \frac{1}{n} \left[\Delta f - (f_{02} + f_{23}) \right] . \qquad (5.25)$$

The time from launch to injection is t_b where

$$t_b = t_{02} + t_{23} + t_c . \qquad (5.26)$$

The longitude at injection is

$$\lambda_i = \alpha_i - \alpha_L - \omega t_b + \lambda_L, \tag{5.27}$$

where α_i is the right ascension of injection which can be obtained from (5.17). The time of injection measured from the midnight of the launch date is

$$t_i = \frac{\alpha_i - \lambda_i - \theta_{go}}{\omega_e} \tag{5.28}$$

where θ_{go} is the sidereal time of Greenwich at zero hours on the launch day. The launch time is then simply

$$t_L = t_i - t_b$$

The final output is t_L and t_c.

As a word of caution, we note that the solution to Ω to exist,

$\dfrac{W_z}{\sqrt{1 - W_z^2}} \; \dfrac{S_z}{\sqrt{1 - S_z^2}}$ has to be less than one. One may easily show that

$1 - S_z^2 - W_z^2 \geq 0$ or that $\cos^2 \delta_S \geq \cos^2 i$ where i is the inclination of the conic plane. This becomes imaginary if $\delta_S > i$. The upper limit exists since specifying the launch azimuth and launch site completely determines the orbit plane inclination from

$$\cos i = \cos \delta_L \sin A_L .$$

Any $\delta_S < i$ can be achieved, while $\delta_S > i$ is forbidden. If the final δ_S is greater than i, the program will halt and print "search routine fails."

APPENDIX 6 CHANGING PHASE AND EPOCH

In this section, we will consider the calculations needed in changing from one conic phase to another. In the least squares computations, the natural parameters to be estimated are the cartesian coordinates of the spacecraft at the beginning of each phase because the radar derivatives for the data within the phase are computed with respect to these coordinates. This is not a restriction, of course, since the final normal matrix can be transformed from one time point to another by means of the variational matrix between the two points.

An essentially identical problem is that of shifting the epoch (which may or may not involve a phase change at the same time). It is known that the variational matrix which occurs as a factor in the radar derivatives becomes increasingly difficult to invert as the separation between the data and the initial epoch becomes larger and larger. This matrix has determinant unity and some elements which grow approximately linearly with time [see Reference 7]. One of the ways to avoid the numerical problems encountered is to advance the epoch so that the times of observations relative to the initial epoch are less than some preassigned number. Advancing the epoch involves essentially the same procedure as changing phase and the latter will be dealt with first leaving any differences to the end.

The simulation of an actual midcourse maneuver is similar to shifting the epoch except for the computation and addition of the maneuver errors.

Orbit Computations in Changing Phase

In approximating the orbit by a sequence of conics, it is implied that there are boundaries at which the center of force should be changed from one body to another. These boundaries are rather nebulous at best. In the present case, we will define them by the use of the sphere of action concept.

The sphere of action about an attracting center of mass, μ_1, in the presence of a larger mass, μ_2, was defined by Yegorov [Reference 8]. Referring to Figure 6, let the two masses be separated by a unit distance and let ρ be the radius of the sphere of action about μ_1; then ρ is obtained

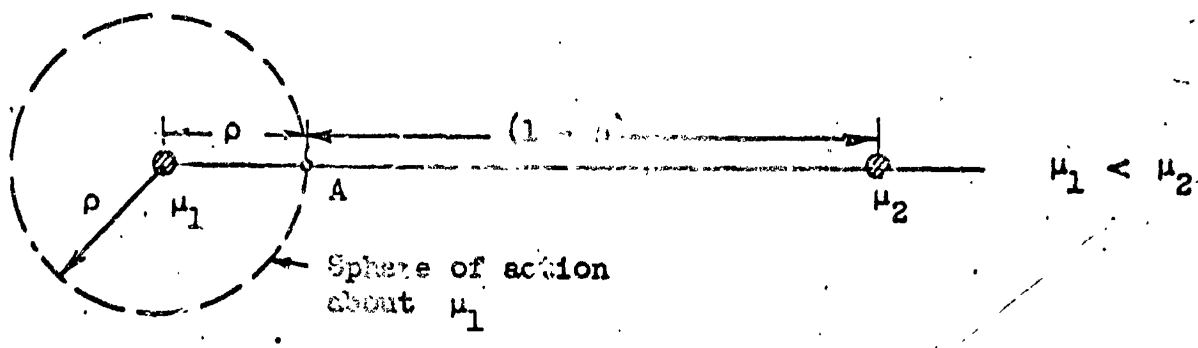

FIGURE 6. Sphere of Action

from the equation

$$\frac{\left[\dfrac{\mu_1}{\rho^2}\right]}{\left[\dfrac{\mu_2}{(1-\rho)^2} - \mu_2\right]} = \frac{\left[\dfrac{\mu_2}{(1-\rho)^2}\right]}{\left[\dfrac{\mu_1}{\rho^2} - \mu_1\right]} \qquad (6.1)$$

If the particle distance to μ_1 is less than ρ, it is in the action sphere of μ_1 and its orbit will be represented by a "1" conic. Conversely, if it is further away than ρ, its orbit will be approximated by a "2" conic. Note that in the left side of (6.1), $\dfrac{\mu_1}{\rho^2}$ is the magnitude of the "central" force due to μ_1 on a particle at "A" and $\left[\dfrac{\mu_2}{(1-\rho)^2} - \mu_2\right]$ is the perturbing force at A due to μ_2 if μ_1 is the primary center. The right side of Equation (6.1) has a similar interpretation. Solving for ρ from (6.1) with the assumption that $\rho << 1$, we find

$$\rho \simeq \left[\frac{\mu_1}{\sqrt{2}\,\mu_2} \right]^{2/5} \tag{6.2}$$

As stated earlier, the boundary at which to change from one center to another is not especially critical. The orbits themselves are somewhat sensitive to the exact values of ρ but the derivatives are in general much less so. In any event, (6.2) serves as a definitive criterion for the sake of consistency. The table below lists the values of ρ being used with the various masses.

μ_1	μ_2	$(\mu_1/\mu_2)^{-1}$	ρ
moon	earth	81.335	.150
earth	sun	332951.3	.00538
Venus	sun	408645	.00496
Mars	sun	3088000	.00221

Having defined the shift boundaries, we next give a rather brief description of the process for finding the shift times. It is assumed that the elements of the conics under consideration are known. There are essentially two types of orbits to consider.

First if the vehicle is departing from the smaller mass into the realm of the larger mass on an escape hyperbola, the eccentric anomaly E_1, at the distance ρ from μ_1 is

$$E_1 = \cosh^{-1} \left(1 - \frac{\rho}{a}\right) \frac{1}{e}$$

where a, e are the usual semi-axis and eccentricity. The time of crossing the sphere can now be found with the aid of Kepler's Equation.

$$t - t_o = \frac{a^{3/2}}{\sqrt{\mu}} \left[E_1 - E_o - e\,(\sinh E_1 - \sinh E_o)\right].$$

t_o and E_o are assumed known.

In the second case, the particle is on a conic about the larger mass which is a transfer arc between two smaller masses. We seek the time at which the spacecraft pierces the target sphere. To be specific consider a helio-centric transfer conic from earth to Mars. As a first approximation the eccentric anomaly at the Mars end is given by

$$E_1 = \cos^{-1} \left(1 - \frac{a_m}{a}\right) \frac{1}{e}$$

where a_m is the semi-major axis of the Mars orbit (due to considerations being taken for quadrants and the number of crossings of the Mars orbit by the spacecraft orbit). The time lapse from the earth end of the heliocentric arc is again found by means of Kepler's Equation. (If a_m is larger than aphelion of the vehicle, then E_1 is set to 180°. A similar test is made if the target body is an inner planet.) This is not the time to begin the planet phase, however. To find the actual shift time, a sequence of E values about E_1 are tried and the distance to Mars calculated at each point. An interpolation (5th order) is then used to find the instant at which the spacecraft pierces the target's sphere. If the orbit misses by such a large amount that it does not hit the sphere, the interpolation stops at the point of closest approach to Mars. From here, the search routine may be used to home in if one chooses.

Having found the shift times, it is a straight forward matter to compute the orbit and variational matrices from the given initial conditions.

Least Squares Computations

The least squares computations in each phase consist essentially of three parts. These are

a) Processing of the radar derivatives to obtain the elements of the normal matrix (A'A).

b) Computing the accuracy criterion (Λ_0).

c) Finding the covariance matrix (Γ) of the orbit parameters in the next phase resulting from data in the current phase. Γ is needed to find the combined estimate due to all the data after changing to the new phase.

Forming the elements of A'A is a rather straightforward summation process on the radar partial derivatives. There are no alterations in this part from one phase to another.

The second and third part will be discussed by considering some particular examples. In general Λ_0 is given approximately by (6.17) and Γ by (6.18). These two expressions contain terms which arise out of the presence of certain physical constants whose values are not to be estimated from the data but whose errors introduce additional terms into the covariance matrix of quantities which are being estimated. If one were to neglect the effects of these constants, Λ_0 and Γ are given simply by (6.5) and (6.6) respectively.

The constants which introduce the added complexity may be classed as dynamical and non-dynamical. The former ones affect the observations by their effects on the actual orbit itself. (For example, gravitational constants). The latter ones affect only the observations and degrade the predictions of orbital variables indirectly by their influence on the initial conditions. The non-dynamical constants usually carry over from phase to phase while dynamical ones do not. For instance, a gravitational constant is dropped in all phases except the one in which it predominates.

We begin by defining the symbols

t_i - initial epoch of the ith phase (epoch); $i = 0, 1, 2 \ldots\ldots$

x_i - initial estimate of parameters to be estimated at t_i.

(r_i, v_i) - initial position and velocity vector at t_i.

Γ_i - a priori covariance matrix of x_i. We assume that Γ_o is given.

p_i - physical constants which are not being improved at t_i.

Λ_{pi} - a priori covariance matrix of p_i.

R_i - data between t_i and t_{i+1}.

N_i - product of the diagonal matrix of variances for ith data times the weighting matrix for the ith data

Λ_i - covariance matrix of the combined estimate of the true values $\hat{x}_{it}$ at t_i.

Λ_{bi} - covariance matrix of impact vector, $b = b\,(x_i,\, p_i)$

μ_e, μ_s - mass of the earth and sun respectively.

$\sigma_{\mu_e}, \sigma_{\mu_s}$ - standard deviations of μ_e, μ_s respectively.

c_o - non-dynamical physical constants.

For the particular values $i = 0, 1, 2$ we define

$$A_o = \left(\frac{\partial R_o}{\partial x_o}\right) \qquad A_1 = \left(\frac{\partial R_1}{\partial x_1}\right)$$

$$P_o = \left(\frac{\partial R_o}{\partial p_o}\right) \qquad P_1 = \left(\frac{\partial R_1}{\partial p_1}\right)$$

A_i and P_i have included in them the factor $\sqrt{W_i}$ where $\sqrt{W_i}$ is the weighting matrix for the ith data, i. e. $\sqrt{W_i}A_i$ is replaced by A_i

$$\lambda_o = \left(\frac{\partial b}{\partial x_o}\right) \qquad \lambda_1 = \left(\frac{\partial b}{\partial x_1}\right)$$

$$\mu_0 = \left(\frac{\partial b}{\partial p_0}\right) \qquad \mu_1 = \left(\frac{\partial b}{\partial p_1}\right)$$

$$\nu_0 = -K_0 A_0' P_0 \qquad \nu_1 = -K_1 A_1' P_1$$

$$K_0 = (A_0' A_0 + \Gamma_0^{-1})^{-1} \qquad K_1 = (A_1' A_1 + \Gamma_1^{-1})^{-1}$$

$$J_0 = A_0' N_0 A_0 + \Gamma_0^{-1} \qquad J_1 = A_1' N_1 A_1 + \Gamma_1^{-1}$$

$$\alpha_0 = \left(\frac{\partial x_1}{\partial x_0}\right) \qquad \alpha_1 = \left(\frac{\partial x_2}{\partial x_1}\right)$$

$$\beta_0 = \left(\frac{\partial x_1}{\partial p_0}\right) \qquad \beta_1 = \left(\frac{\partial x_2}{\partial p_1}\right)$$

We will use a shift from an earth to a sun centered conic as an illustration of changing phase. The different cases indicate the simplifications which result from (6.17) and (6.18) depending on the elements of the vectors x_i and p_i. Hence the cases are classified according to the elements in the two vectors.

<u>Case 1</u> $x_0 = (r_0, v_0)$ $\quad p_0 = 0$ ($p = 0$ means p is empty)

$x_1 = (r_1, v_1)$ $\quad p_1 = 0$

At $t = t_0$; compute J_0 and K_0 as given above from A_0; define L_0 by

$$L_0 = K_0 J_0 K_0 \tag{6.3}$$

$$\Lambda_c = L_0 \tag{6.4}$$

$$\Lambda_{bo} = \lambda_0 \Lambda_0 \lambda_0' \tag{6.5}$$

Updating to t_1 requires $x_1 = x_1(x_o)$ and

$$\Gamma_1 = \alpha_o \Lambda_o \alpha_o' \quad . \tag{6.6}$$

At t_1, find L_1 from

$$L_1 = K_1 J_1 K_1 \tag{6.7}$$

$$\Lambda_1 = L_1 \tag{6.8}$$

$$\Lambda_{b1} = \lambda_1 \Lambda_1 \lambda_1' \tag{6.9}$$

<u>Case 2</u> $x_o = (r_o, v_o, \mu_e) \qquad p_o = 0$

$x_1 = (r_1, v_1) \qquad p_1 = 0$

This is formally identical to Case (1). A_o has an added column, $\left(\frac{\partial R}{\partial \mu_e}\right)$; L_o is 7 x 7 instead of 6 x 6.

In equation (6.6), α_o becomes a 6 x 7 matrix instead of 6 x 6; the last column is $\left(\frac{\partial x_1}{\partial \mu_e}\right)$.

Case 3 $\quad x_o = (r_o, \cdot\,)$ $\qquad p_o =$

$x_1 = (r_1, v_1, \mu_s)$ $\qquad p_1 = 0$

Again this is formally the same as Case 1. $A_1'A_1$ is now 7 x 7, Γ_1 is 7 x 7 instead of 6 x 6, and we have

$$(\Gamma_1)_7 = \left(\begin{array}{c|c} (\Gamma_1)_6 & 0 \\ \hline 0 & \sigma^2_{\mu_s} \end{array}\right) \tag{6.10}$$

The off diagonal elements of (6.10) would not be zero if x_1 is the initial point in the sun phase because the astronomical unit uncertainties would introduce errors into x_1. However if x_1 was the last point in the earth phase, then the off diagonals would be zero. For the sake of simplicity the second choice is to be preferred and will be assumed for the remainder of the discussion.

L_1, Λ_1 and Λ_{b1} are formally the same as in Case 1.

Case 4 $\quad x_o = (r_o, v_o, \mu_e)$ $\qquad p_o = 0$

$x_1 = (r_1, v_1, \mu_s)$ $\qquad p_1 = 0$

This is a combination of Case 2 and 3. Hence, no real differences are involved.

Case 5 $\quad x_o = (r_o, v_o, c_o)$ $\qquad p_o = 0$

$x_1 = (r_1, v_1, c_o)$

This case is again the same as Case 1 with the addition of a non-dynamical constant in each phase. α_o is modified to

$$\alpha_{o7} = \left(\frac{\partial x_1}{\partial x_o}\right)_7 = \left(\begin{array}{c:c} \left(\frac{\partial x_1}{\partial x_o}\right)_6 & 0 \\ \hdashline 0 & 1 \end{array}\right)$$

Case 6

$$x_o = (r_o, v_o) \qquad p_o = \mu_e$$

$$x_1 = (r_1, v_1) \qquad p_1 = 0$$

L_o is the same as (6.4); Λ_o and Λ_b are

$$\Lambda_o = L_o + K_o A_o' P_o \Lambda_{po} P_o' A_o K_o \tag{6.11}$$

$$\Lambda_{bo} = \lambda_o L_o \lambda_o' + (\mu_o + \lambda_o \nu_o) \Lambda_p (\mu_o + \lambda_o \nu_o)' \tag{6.12}$$

In this simple case, P_o and Λ_{po} are just

$$P_o = \left(\frac{\partial R_o}{\partial \mu_e}\right)$$

$$\Lambda_{po} = \sigma^2_{\mu_e} .$$

Updating to t_1,

$$\Gamma_1 = \alpha_o L_o \alpha_o' + (\beta + \alpha_c' \nu_o) \Lambda_{po} (\beta + \alpha_o \nu_o)' . \tag{6.13}$$

At t_1,

$$L_1 = K_1 J_1 K_1' = \Lambda_1 \tag{6.14}$$

$$\Lambda_{b1} = \lambda_1 L_1 \lambda_1' \tag{6.15}$$

Case 7

$$x_o = (r_o, v_o) \qquad p_o = \mu_e$$

$$x_1 = (r_1, v_1) \qquad p_1 = \mu_s$$

This is essentially the same as Case 6, L_o, Λ_o, Λ_{bo}, Γ_1 are the same as (6.4), (6.11), (6.12), (6.13). At t_1; L_1 is given by (6.14), Λ_1 and Λ_{b1} are given by (6.11) and (6.12) respectively with subscript "1" in place of "0".

Also at t_1,

$$P_1 = \left(\frac{\partial R_1}{\partial \mu_s}\right)$$

$$\Lambda_{p1} = \sigma^2_{\mu_s} \quad .$$

Case 8

$$x_o = (r_o, v_o) \qquad p_o = (\mu_e, c_o)$$

$$x_1 = (r_1, v_1) \qquad p_1 = (\mu_s, c_o)$$

At t_o, L_o, Λ_o, Λ_b are given by (6.3), (6.11), and (6.12) respectively; Γ_1 is the same as (6.13) with

$$\beta = \begin{pmatrix} \frac{\partial x_1}{\partial \mu_e} \\ 0 \end{pmatrix}$$

$$P_o = \begin{pmatrix} \frac{\partial R_o}{\partial \mu_e} \\ \\ \frac{\partial R_o}{\partial c_o} \end{pmatrix}$$

At t_1, L_1 is the same as (6.14); Λ_1 and Λ_{b1} are

$$\Lambda_1 = L_1 - \xi \Lambda_p \xi_1' + (\nu_1 + \xi_1) \Lambda_p (\nu_1 + \xi_1)' \tag{6.16}$$

$$\Lambda_{b1} = \lambda_1 (L_1 - \xi_1 \Lambda_p \xi_1') \lambda_1 + (\mu_1 + \lambda_1 (\nu_1 + \xi_1)) \Lambda_p (\mu_1 + \lambda_1 (\nu_1 + \xi_1))', \tag{6.17}$$

where ξ_1 is

$$\xi_1 = K_1 \Gamma_1^{-1} (\alpha_o \nu_o + \beta_o) .$$

If the epoch is changed in the sun phase from t_1 to t_2, then Γ_2 is found from

$$\Gamma_2 = \alpha_1 (L_1 - \xi \Lambda_p \xi') \alpha_1' + (\beta_1 + \alpha_1 (\nu_1 + \xi_1)) \Lambda_p (\beta_1 + \alpha_1 (\nu_1 + \xi_1))' \tag{6.18}$$

If t_2 is the time of an actual midcourse maneuver in which the covariance matrix of maneuver errors is Λ_e (See Appendix 7), then the covariance matrix of the vector, x_2, after the maneuver is given by Γ_{2a} where

$$\Gamma_{2a} = \Gamma_2 + \Lambda_e .$$

In general if x_n and p_n are initial estimates in the nth phase, expressions for Λ_{bn} and Γ_{n+1} will contain correlation terms, $C_n = \overline{\delta x_n \; \delta p_n'}$. Formulas for Λ_{bn} and Γ_{n+1} become quite involved after changing epochs a few times. They may be simplified considerably if one is willing to neglect certain of the correlation terms. If in the nth phase one assumes that $C_n = 0$, then (6.12) and (6.13) are valid (with the appropriate subscripts of course). If $C_n \neq 0$ but $C_{n-1} = 0$; then (6.17) and (6.18) hold. No plans exist at the present for including more than the $(n-1)^{st}$ term. This is perhaps justifiable by asserting that the effects due to the constants ought to be quite small in the first place. Otherwise one should be able to improve their values from the data. If the constants are elements of the vector to be improved, then they do not present these problems.

Inversion of Matrices

In much of the foregoing, the final results are obtained after inverting a symmetric matrix. The method which is presently used for these inversions is a recursive formula suggested by Lass and Solloway. [Reference 9]

Given the matrix M

$$M = \begin{pmatrix} A & x \\ x' & a \end{pmatrix}$$

with A^{-1} known, M^{-1} is,

$$M^{-1} = \begin{pmatrix} B & y \\ y & b \end{pmatrix}$$

where

$$W = A^{-1} x \tag{6.19}$$

$$b = (a - x' W)^{-1} \tag{6.20}$$

$$y = -W b \tag{6.21}$$

$$B = A^{-1} - W y' \tag{6.22}$$

For a matrix of order n with elements α_{ij}, start by setting $A_1 = \alpha_{11}$, $A_1^{-1} = \frac{1}{\alpha_{11}}$, $x_1 = \alpha_{12}$, $a_1 = \alpha_{22}$. Substituting into (6.19) through (6.22) yields the 2 x 2 inverse which is then put back into the algorithm to find the 3 x 3 inverse etc., until the n x n inverse is obtained.

APPENDIX 7 MANEUVER ERROR COVARIANCE MATRIX

In the simulation of a hypothetical or an actual maneuver, the covariance matrix of maneuver errors (Λ_e) is added on to the tracking covariance matrix at the maneuver point. This appendix outlines in brief the computational aspects of finding Λ_e (It is not intended to show the formulations leading to the final results. For further details [see Reference 10]).

Let

- t be the maneuver time;
- b the impact vector,
- Λ_b the covariance matrix of b due to tracking prior to t,
- V the spacecraft velocity vector at time t,
- $\left(\frac{\partial b}{\partial V}\right)$ a 3 x 3 matrix of partial derivatives evaluated at t.

The correction vector, δV, is obtained from

$$v = \delta V = Q\delta b$$

$$Q = \left(\frac{\partial b}{\partial V}\right)^{-1} .$$

The covariance matrix of v is

$$\Lambda_v = \overline{v\,v'} = Q\left[\lambda_o \Gamma_o \lambda_o' + \Lambda_b\right] Q'$$

and the ensemble average of the square magnitude is

$$\overline{v^2} = \overline{v'v} = \overline{\delta b' Q' Q \delta b} = \text{Trace}\; Q\left[\lambda_o \Gamma_o \lambda_o' + \Lambda_b\right] Q'$$

The error in the execution of the maneuver results from 4 independent sources. The sources are

(1) shut off errors (subscript s)

(2) resolution errors (subscript r)

(3) pointing errors (subscript p)

(4) autopilot errors (subscript a).

It was shown in Reference [10] that the covariance matrix of maneuver errors (Λ_e) arising from the above sources is given by

$$\Lambda_e = (\sigma_s^2 - \sigma_p^2)\,\Lambda_v + (\sigma_r^2 - \sigma_a^2)\,\Gamma + (\sigma_p^2 \bar{v}^2 + \sigma_a^2)\,I \quad ,$$

where

σ_s^2, σ_r^2, σ_p^2, σ_a^2 are variances of scaler random variables associated with each of the 4 types of errors above, which for our purposes are assumed to be given. (They are required inputs for any midcourse simulation). The exact definitions of these quantities are given in Reference [11];

I is the 3 x 3 identity matrix. The only remaining quantity which has not been defined is Γ. Γ is the matrix $\left(\frac{\overline{v\,v'}}{v^2}\right)$ whose elements are

$$\Gamma = \begin{pmatrix} \frac{v_x^2}{v^2} & \frac{v_x v_y}{v^2} & \frac{v_x v_z}{v^2} \\ & \frac{v_y^2}{v^2} & \frac{v_y v_z}{v^2} \\ (\text{symmetric}) & & \frac{v_z^2}{v^2} \end{pmatrix} .$$

The orthogonal transformation which diagonalizes Λ_V will reduce each element of Γ to linear combinations of integrals each of which has the form

$$I = \frac{2\pi^{-3/2}}{\sigma_\xi \sigma_\eta \sigma_\zeta} \int\!\!\int\!\!\int_{-\infty}^{\infty} \frac{\xi^2}{\xi^2+\eta^2+\zeta^2} \exp - \frac{1}{2}\left[\frac{\xi^2}{\sigma_\xi^2} + \frac{\eta^2}{\sigma_\eta^2} + \frac{\zeta^2}{\sigma_\zeta^2}\right] d\xi d\eta d\zeta \;.$$

The value of this integral depends on the relative magnitudes of σ_ξ, σ_η, σ_ζ. In general the results are expressible in terms of elliptic integrals of the first and second kind, see Reference 12. A subroutine for the evaluation of I, using an elliptic integral subroutine, has been compiled at STL in conjunction with TAPP.

APPENDIX 8 COMPUTATION OF WEIGHTS

The rules for computing the final least squares weights will be described in this appendix. These rules were specified by T. W. Hamilton [Reference 13] for use with the Deep Space Instrumentation Facility but are easily adaptable to other applications.

The weighting matrix, W, appears in the normal matrix, A'WA. For brevity, it has been conveniently absorbed into the A matrix in most of the writing (i.e., we replace A with $\sqrt{W}A$). For our purposes, we need to find both W and its product with M, the diagonal matrix of noise variances. W itself is a function of M. Hence we indicate the procedure in the program for computing the elements of M and the formula for finding the final weights from it.

The elements of M are sums of variances from independent noise sources. These sources are tabulated in Table 4 for the different types of data from a typical earth based tracker. If the noise from a particular source is correlated with a correlation time greater than the sampling interv l, then the corresponding variance is degraded by the scale factor τ/s where τ is the correlation time and s the sampling interval.* For a given data type, the total variance is given by the sum.

$$\sigma^2 = \sum_i \sigma_i^2 + \sum_j \sigma_j^2 \tau_j/s \qquad (8.1)$$

where the sum on i is over all sources such that $\tau_i \leq s$ and the sum on j is over sources with $\tau_j > s$.

* See Reference [14] for details of replacing correlated noise with "equivalent-or-worse" uncorrelated noise.

The final σ's may or may not be constants. For example in the measurement of the two angles in a polar coordinate system such as azimuth and elevation or hour angle and declination, we have

$$\sigma_{\phi} = \frac{\sigma}{\cos\theta} \tag{8.2}$$

where

ϕ is the azimuthal or longitudinal angle

θ is the polar or cone angle

σ is the standard deviation of an angular measurement at $\theta = 0$ (azimuthal or equatorial plane). σ is calculated from (8.1)

Since $\sigma_{\phi} \rightarrow \infty$ as $\theta \rightarrow 90°$, measurements of ϕ are weighted much less in the region where ϕ becomes ill-defined.

As a second example, if the ground based oscillator frequency drifts at a rate $\dot{f}$ in the measurement of two way doppler, then the drift causes an error in the measured doppler over the light transit time interval of

$$\delta f = 2\dot{f}\,\frac{R}{C}$$

where C is the velocity of light and R is the slant range. The corresponding range rate error is

$$\delta\dot{R} = \frac{1}{2}\,\frac{C}{f_0}\,\delta f = \left(\frac{\dot{f}}{f}\right) R \sim \sigma_{\dot{R}}$$

In this case, the standard deviation increases with the range.

The variance formulas for a spacecraft based tracker are listed below for each individual type of observation.

1) Clock Angle (A) and Cone Angle (B)

$$\sigma_A^2 = \left[\sigma_j^2 + \sigma_{ro}^2 + (K_A\ C)^2\right] \frac{1}{\cos^2 B} \qquad (8.3)$$

$$\sigma_B^2 = \left[\sigma_j^2 + \sigma_{ro}^2 + (K_B\ C)^2\right] \qquad (8.4)$$

where

σ_j^2 - variance due to electrical jitter

σ_{ro}^2 - variance due to readout

K_A, K_B - adjustable constants

C - planetary angular diemeter from the spacecraft.

2) Occultations (α, δ)

$$\sigma_\alpha^2 = \left[\sigma_j^2 + (K_\alpha\ C)^2\right] \frac{1}{\cos^2 \delta} \qquad (8.5)$$

$$\sigma_\delta^2 = \left[\sigma_j^2 + (K_\delta\ C)^2\right] \qquad (8.6)$$

3) Planetary Angular Diameter (C)

$$\sigma_C^2 = (K_C\, C)^2 \tag{8.7}$$

The least squares weight to be used in the normal matrix is

$$W_K = (\omega_K^2 + f_K\, \sigma_K^2)^{-1} \tag{8.8}$$

where σ_K^2 is typically obtained from (8.1) through (8.7) for the various types of data. ω_K^2 and f_K are adjustable input constants for the individual data types.

In addition to W, the matrix WM is needed to find the weighted least squares covariance matrix, from (8.8) the elements of WM are

$$(WM)_K = \frac{\sigma_K^{\ 2}}{\omega_K^{\ 2} + f_k\, \sigma_K^{\ 2}} . \tag{8.9}$$

Table 4. Typical Noise Model for Terrestrial Tracker

Data Type	Noise Source	σ (Typical Values)	τ (Typical Values)
angular measurement (each type)	readout error	$.003^{\circ}$	10 seconds
	angenna deflections	$.007^{\circ}$	5 hours
	jitter	$.01^{\circ}$	10 seconds
doppler shift	oscillator drift	$\left(\frac{\dot{f}}{f}\right)_T R$	T
	round off	$\frac{C}{2\sqrt{3}\, fT}$	T
	system error	$\delta\dot{R}$	10 seconds
range	clock error	$K_C\ R$	1 hour
	system error	5 meters	5 hours
	round off	10 meters	10 seconds

R is the slant range

T is the counting interval = .01 kilosecond unless otherwise specified

$\left(\frac{\dot{f}}{f}\right)_T$ is the percentage drift rate = 81×10^{-10} per kiloseconds

C is the velocity of light = 2.99795×10^{5} km/sec

f is the oscilator frequency = 9×10^{12} cycles per kilosecond

K_C is a constant = 10^{-10}

$\delta\dot{R}$ is velocity = 1.0×10^{-4} km/sec unless specified otherwise.

APPENDIX 9 OUTPUT FORMATS

This appendix describes the formats for the various output print routines. These are:

(I) The input print

(II) Change of phase print

(III) Rise-set print

(IV) Coarse accuracy print

(V) Fine accuracy print

(VI) Trajectory print (the format for the trajectory print is given in appendix 10 since it contains a considerable number of independently defined quantities).

In its present form TAPP has five output options, selected by input flags, and consisting of different combinations of the printout lists I to VI*. The options and their contents are:

a) Trajectory only

List I; items 1, 2, 3, 4, 10

List VI

(The contents of the numbered items are given in the list)

b) Rise-set only

List I; items 1, 2, 3, 5, 6, 10

List III

c) Rise-set plus trajectory

List I; items 1, 2, 3, 4, 5, 6, 10

List III, VI

---

* A special midcourse analysis output due to C. Pfeiffer of JPL has been requested and will be incorporated as a sixth printout option.

d) <u>Coarse accuracy</u>

(no midcourse maneuvers permitted)

List I - all applicable items

Lists II, III, and IV

List VI at option

e) <u>Fine accuracy</u>

(midcourse permitted)

List I - all applicable items

Lists II, III, and V

Lists VI at option

An inspection of list (I) will help in understanding the input requirements. We now give the format and contents of the lists I through VI along with explanatory notes for each. Typical quantities will be inserted in blank spaces where pertinent.

I. INPUT PRINT FORMAT

(all words in capital letters are printed on the output)

Items

(1) GD 59 1 1 00 00 00.00 JD 2436569.5

PHASE SEQUENCE earth sun Mars

(2) INITIAL CONDITIONS earth CENTERED equatorial PLANE

x	$\dot{x}$	r	V
y	$\dot{y}$	δ	Γ
z	$\dot{z}$	α	Σ

(2a) V-INFINITY INPUT

C_3	A_L	t_{02}	t_c
δ_s	Φ_L	f_{02}	t_L
α_s	λ_L	t_{23}	
	Γ	f_{23}	
	R		

(3) TARGET CONDITIONS

m_1 t_f (days)

m_2

SEARCH CONDITIONS

SEARCH ON initial conditions

COMPONENTS V

Γ

Σ

SCALE FACTORS

1	4
10	3
5	2

(4) PRINT INTERVALS

STRT	STOP	SPACING
0	.5	60
.5	100	14400
100	110	180

(5) TRACKING STATIONS

STN	CODE	LONG	LAT	ALT	LIM-I	LAM-I	MOD-I
Gldjet	3	-120	34	.5E-4	1	2	3

(6) TRACKING SCHEDULE

STN	INTL	TYPE	D-STRT	WAIT-1	D-STOP	WAIT-2	DLTA-D	SPACNG
3	1		0		5		1	
		3		120		120		1200
		4		120		120		1200
	2		7		100		3	
		6		3600		4000		7200

(6a) SPACECRAFT OBSERVATIONS

TYPE	D-STRT	D-STOP	SPCNG	PLNT	STAR
A	40	100	5	VENUS	CANOPUS
B	40	100	5	VENUS	CANOPUS
C	OPTION	2			
D	20	40	2	VENUS	
E	20	40	2	VENUS	

TYPE C WILL START WHEN (OPTION 2)

R LESS THAN 4.0

SPCNG 5 10 15 25 30 40 50

(7) ADJUSTABLE PARAMETERS — FIXED PARAMETERS

$x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z} \quad \mu_e$ — $\mu_s \quad c$

DATA BIAS

STN	TYPE	D-STRT	D-STP
3	3	0	1

(8) A PRIORI KNOWLEDGE

UNCORRELATED	VARIANCE	UNCORRELATED	VARIANCE
μ_e	.1 E-9	μ_s	.1 E-8
		c	.1 E-11

CORRELATED

GROUP 1 x y z (for example)

Γ_o (x, y, z)

GROUP 2 $\dot{x}$ $\dot{y}$ $\dot{z}$ (for example)

Γ_o ($\dot{x}$ $\dot{y}$ $\dot{z}$)

(9) LEAST SQUARES WEIGHTS

Option

STN	TYPE	F	OMGA	SGMA
3	3	1	0	.5
	4	1	0	.5
	6	0	1.0	1.0

SPACECRAFT OBSERVATION WEIGHTS

	A	1	0	.05
	D	1	0	.05
	E	1	0	.05

(10) CONTROLS AND OPTIONS

TYPE INTERCEPT	2		(IPF only)
STEP SIZE CONTROL	ϕ_1	ΔE_1	
	ϕ_2	ΔE_2	
OUTPUT OPTION	E		SLICES 2

PRINT INITIAL CONDITION COVARIANCE - no

INPUT ELEMENTS

BODY	a	e	i	Ω	ω	M_o
J.D.	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$

(11) MANEUVER PRINT

HYPOTHETICAL	D-STRT	D-STP	SPCNG		
	20	100	10		
ACTUAL	65D.	12H	20M	14.50S	FROM EPOCH

S-S

S-R

S-P

S-A

COMPONENTS 2

Explanatory Notes to Input Print Format

All units used in the initial printout will be in megameters for length; kilometers/second for velocity; kiloseconds for time and degrees for angles unless otherwise noted.

(1) CD - Gregorian Date of epoch - year month day hour minutes seconds GMT

JD - Julian Date of epoch - days

PHASE SEQUENCE - sequence of centers for trajectory

(2) Initial Conditions - position and velocity at epoch with respect to the given primary center and reference plane. Both Cartesian and polar coordinates are printed.

(2a) Items in (2a) will be printed only if the initial conditions are given in terms of velocity at infinity on a geocentric escape hyperbola.

C_3 - (hyperbolic excess velocity)^2 = twice vis-viva energy

δ_s - declination of asymptote

α_s - right ascension of asymptote

A_L - launch azimuth

δ_L - launch declination

λ_L - launch longitude

See appendix (5) for the remaining quantities.

(3) Target Conditions and Search Conditions

Items in (3) will be printed only if the option to search has been elected.

m_1, m_2 - miss components

t_f - total flight time

SEARCH ON - may be initial conditions or velocity vector at infinity (the latter on geocentric escape only)

COMPONENTS

The components to be varied will be printed if they are specified. Otherwise the machine finds the search parameters with the "smallest" magnitude of change. The number of components specified must agree with the number of target conditions prescribed (2 or 3). If there are two, they may be

2 components out of 3 of V_∞.

2 components out of 6 of initial conditions (polar or Cartesian).

If there are three target conditions, all 3 elements of V_∞ must be varied, but one can still choose any 3 elements of the 6 initial conditions.

SCALE FACTORS

If the machine finds the parameters to change, it will select (2 or 3) quantities such that the magnitude of the correction vector is the smallest. The scale factors (dimensionless) are to allow for changing the relative weights on the independent variables (see appendix 5).

(4) Print Intervals

Items in (4) are for use with "trajectory print only" cases. A sequence of start, stop time (days) and spacing (minutes) are given.

(5) Tracking Stations

STN - name of station

CODE - 2 digit identification number for station

LONG, LAT, ALT - longitude, latitude, altitude.

LIM-I - 1 digit code to select from 10 station limitation models stored in program. Each table specifies a minimum elevation angle and a maximum range for the tracker.

LAM-I - 1 digit code which selects station location uncertainty model (3 x 3 diagonal matrix) from a total of 10.

MOD-I - 1 digit code which selects station radar noise model from a total of 5 stored in the program.

(6) Tracking Schedule

STN - Station code, see (5)

INTL - Time interval given by D-STRT and D-STOP

TYPE - Type of radar observation identified as follows:

Number	Type
1	hour angle
2	declination
3	elevation
4	azimuth
5	two way doppler
9	slant range

D-STRT - first day of the interval given by number under INTL (1 in this case) measured in days from initial epoch.

WAIT-1 - wait time between rise and beginning of data on each tracking day. (minutes)

D-STOP - last day of interval in days from initial epoch.

WAIT-2 - time in minutes before set on each tracking day to cease tracking.

DLTA-D - number of days between tracking days.

SPACNG - time between data points. (seconds)

(6a) Items (6a) are printed if there are to be spacecraft observations.

TYPE - types of observation are identified by the letters

Letter	Type
A	clock angle
B	cone angle
C	planetary diameter
D	occultation declination
E	occultation right ascension

D-STRT - days from initial epoch to take first observation.

D-STOP - days from initial epoch to take last observation.

SPCNG - days between each observation.

PLNT - reference planet for spacecraft observations. (See Appendix 3)

STAR - reference star for spacecraft observations. Canopus or input.

TYPE C WILL START WHEN - In planetary diameter measurements the start time may be specified as time when distance of spacecraft to planet is less than some value R. (in megameters)

SPCNG - times of C observations after the first observation (hours).

(7) Adjustable parameters are the elements of the vector to be improved. If the elements include data biases, the observations containing the bias are specified by:

STN - Station code number.

TYPE - Type data - see note 6 for correspondence.

D-STRT - Start day of observation - days from epoch.

D-STP - Stop day of observation - days from epoch

(8) A priori knowledge

The adjustable and fixed parameters are divided into groups such that the groups are uncorrelated with each other. The variances and covariances are given with the following units:

x y z -	megameters
$\dot{x}$ $\dot{y}$ $\dot{z}$ -	kilometers/second
mass constant -	percentage error
velocity of light -	percentage error
station coordinates-	degrees for angles, megameters for altitude.

(9) Least Squares Weights

f and ω are defined in Appendix 8.

SGMA - The final standard deviation obtained from the noise model

Similar definitions hold for the quantities under spacecraft observations.

(10) Controls and Options

TYPE INTERCEPT - 1 digit number giving number of crossings of target orbit at estimated time of intercept.

STEP SIZE - eccentric anomaly increments taken in each phase in computing orbit

OUTPUT OPTION - may be any one of options (a) to (e), if (e) is selected, it is necessary to specify SLICES, a number which subdivides a tracking day into intervals for print purposes (see note V(1) below).

PRINT INITIAL CONDITION COVARIANCE - The covariance matrix of adjustable parameters will not be printed unless requested by option.

INPUT IS V-INFINITY - the driver and the constraints going with it must be specified. (See Appendix 5.)

INPUT ELEMENTS - the osculating elements (classical or position and velocity at a given time, JD) may be read into the program replacing the constants which are tabulated in Appendix 2.

(11) Maneuver Print

Items in (11) will be printed if there are hypothetical or actual maneuvers (only one actual maneuver permitted).

HYPOTHETICAL D-STRT D-STP SPCNG - specifies the start time, stop time and spacing of the hypothetical maneuvers. (units are days for all three).

ACTUAL - time in days, hours, minutes, and seconds from epoch for epoch for actual maneuver.

S-S, S-R, S-P, S-A - standard deviations associated with shut off, resolution, pointing, and autopilot error constants respectively.

COMPONENTS - correct 2 or 3 components.

II. CHANGE OF PHASE FORMAT

(1) GD JD

(2) ELEMENTS WRT NEW CENTER

x	$\dot{x}$	a	Ω
y	$\dot{y}$	e	ω
	$\dot{z}$	i	M_o

(3) SPCT WRT OLD OLD WRT NEW

x	$\dot{x}$	x	$\dot{x}$
y	$\dot{y}$	y	$\dot{y}$
z	$\dot{z}$	z	$\dot{z}$

(4) Λ_{bo} Λ_{bn}

(5) Λ_{xo}

Λ_{xn}

Explanatory Notes to Change of Phase Format

(1) GD - Gregorian date - year month day hour min sec GMT

JD - Julian date-days

(2) ELEMENTS WRT NEW CENTER

Cartesian coordinates at epoch

Classical elements at epoch

(3) SPCT WRT OLD - space craft Cartesian coordinates with respect to old center.

OLD WRT NEW - old center Cartesian coordinates with respect to new center.

(4) Λ_{bo}, Λ_{bn} - covariance matrix of target parameters before and after changing phase.

(5) Λ_{xo}, Λ_{xn} - covariance matrix of Cartesian coordinates before and after changing phase.

III. RISE-SET FORMAT

YEAR					
STN	RISE	T-TO	SET	T-TO	EMAX
3	12 14 13 41 15.6	43.2108	12 15 01 41 15.6	43.7108	54.82

Notes:

Rise-set times for all stations are tabulated in chronological order using the format above. The day interval is given by DLTA-D unless otherwise specified.

RISE - rise time in month day hours minutes seconds.

T-TO - time from epoch in days.

EMAX - maximum elevation angle during pass.

IV. COARSE ACCURACY FORMAT

(1) GD T-TO
STN 3 TYPE 3, 4 RISE or SET

(2)

SPORT	WRT	MAIN	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
EARTH	WRT	SUN	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
TARGET	WRT	SUN	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$

(3) Λ_b LAM 1
LAM 2
THETA

(4) Λ_o

Notes:

(1) GD - Date

T-TO - time in days from epoch

In the coarse print, the output occurs at the beginning and end of each type of station.

STN designates the station, TYPE the data type, RISE or SET tells whether the prints go with the beginning or end of a pass.

(3) Λ_b - target covariance matrix

LAM 1, LAM 2 - eigenvalues of upper left 2 x 2 in Λ_b.

THETA - angle between major axis of dispersion ellipse in m_1, m_2 plane and the m_1 axis.

(4) Λ_o - initial condition covariance matrix - at option only.

V. FINE ACCURACY FORMAT

(1) GD T-TO

SLICE - 3

(2) - (4) same as coarse format

(5) MANEUVER

MIJ h_1 h_2 h_3

e_{1x} e_{2x} e_{3x}
e_{1y} e_{2y} e_{3y}
e_{1z} e_{2z} e_{3z}

NON-CRTCL DRCTN

n_x
n_y
n_z

Λ_v Λ_e

$\Lambda_b + \Lambda_{ba}$

Notes on Fine Accuracy Format:

(1) GD - Date

T-TO - time from epoch (days)

GD and T-TO will refer to the time of the maneuver if the print time is a maneuver time.

SLICE - Each tracking day will be divided into N slices. (intervals)

The fine output is printed at each dividing point except the very first.

The fine output is also printed out at each maneuver point (hypothetical or real).

(2) - (4) These items are the same as in the coarse format.

(5) MANEUVER - the items in this group are printed at the maneuver points. Maneuvers can only be requested along with the fine printout.

MIJ - the impact vector sensitivity matrix $\left(\frac{\partial b}{\partial v}\right)$ at the maneuver time (see Appendix 4)

h_1, h_2, h_3 - magnitudes of the rows of MIJ.

e_{ix}, e_{iy}, e_{iz} - unit vectors having the same direction as the rows of MIJ.

NON CRTCL DRCTN n_x, n_y, n_z - direction cosines of the non-critical direction.

Λ_v - covariance matrix of required velocity.

Λ_e - covariance matrix of maneuver errors (Appendix 7)

$\Lambda_b + \Lambda_{ba}$ - covariance matrix of target parameters after the maneuver.

APPENDIX 10 TRAJECTORY PRINTOUT

In this appendix, we give a key for the quantitites which are printed if a trajectory alone is to be computed. Following the key, the symbols are defined and the formulas for computations are either given or referenced to some other section of this report. All units will be in megameters for length, kilometers per second for velocity, and degrees for angles (unless otherwise stated).

The printout along the trajectory will depend on the phase. However, the initial print in each phase will be the same.

I. Print Key

Initial Print in each Phase:

(a)	GGD		JD	ϕ	RP	TC
(b)	a	e	i	Ω	ω	M_o
(c)	p	τ				
(d)	C_3	C_1	qq	f	Q	P
(e)	$V_{\infty x}$	$V_{\infty y}$	$V_{\infty z}$	α_s	δ_s	

Printout along the trajectory:

Geocentric Phase

$(t-t_o)_{ks}$ $(t-t_o)_d$

(1)	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
(2)	r	δ	α	V-	Γ	Σ
(3)	r	ϕ	Θ	V	γ	σ

Heliocentric Phase

$(t-t_o)_{ks}$ $(t-t_o)_d$

(1)	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	geocentric
(2)	r	δ	α	V	Γ	Σ	
(3)	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	heliocentri
(4)	r	β	λ	V	Γ	Σ	
(5)	EPS	EPM	SPC	TPC	SPT	STP	
(6)	LOE	LOT	EMP				

Target Phase

(a)	$\bar{B}.\bar{T}$	$\bar{B}.\bar{R}$	B	t_f
(b)	M_{11}	M_{12}	M_{13}	
(c)	M_{21}	M_{22}	M_{23}	
(d)	M_{31}	M_{32}	M_{33}	

$(t-t_o)_{ks}$ $(t-t_o)_d$

(1)	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	geocentric
(2)	r	δ	α	V	Γ	Σ	
(3)	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	targetcentr
(4)	r	β	λ	V	Γ	Σ	
(5)-(6)	same as in heliocentric phase						

II. Definitions

Initial Print:

line, col.	Symbol
a1	GD - Geogorian Date (Year Month Day Hour Min Sec GMT)
a2	JD - Julian Date (days)
a3	ϕ - name of primary center
a4	RP - reference plane - ecliptic or equatorial
a5	TC - type conic - elliptic or hyperbolic
b1-b6	a, e, i, Ω, ω, M_o - classical elements
c1	p - semi latus rectum
c2	τ - time of perigee passage
d1	C_3 - twice vis-viva energy
d2	C_1 - angular momentum
d3	q - distance of closest approach
d4	f - true anomaly
d5	Q - $a(1 + e)$
d6	P - period
	Line e is printed only if TC is hyperbolic
e1-e3	$V_{\infty x}$, $V_{\infty y}$, $V_{\infty z}$ - components of V_∞
e4	α_s right ascension of V_∞
e5	δ_s declination of V_∞

Geocentric Phase

$(t-t_o)_{ks}$, $(t-t_o)_d$ - time from epoch in kiloseconds and days

11-16 $x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}$ - goecentric equatorial coordinates

21-26 $r\ \delta\ \alpha\ V\ \Gamma\ \Sigma$ - inertial geocentric polar coordinates (equatorial)

31-36 $r\ \phi\ \theta\ V\ \gamma\ \sigma$ - earth fixed geocentric polar coordinates

Heliocentric Phase

11-16 $x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}$ - geocentric ecliptic coordinates

21-26 $r\ \delta\ \alpha\ V\ \Gamma\ \Sigma$ - same as 21-26 in geocentric phase

31-36 $x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}$ - heliocentric ecliptic coordinates

41-46 $r\ \beta\ \lambda\ V\ \Gamma\ \Sigma$ - heliocentric polar coordinates (ecliptic)

51-63 These are celestial angles which are defined in the computational formulas

Target Phase

a1 $\bar{B}.\bar{T}$ - component of miss in the ecliptic (impact parameter)

a2 $\bar{B}.\bar{R}$ - component of miss in B plane normal to ecliptic

a3 B - magnitude of miss

a4 t_f - total flight time

b1-d3 M_{ij} - miss coefficients matrix elements defined below in Section III

11-16 $x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}$ - geocentric ecliptic coordinates

21-26 $r\ \delta\ \alpha\ V\ \Gamma\ \Sigma$ - inertial geocentric polar coordinates (equatorial)

31-36 $x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}$ - target centered ecliptic coordinates

41-46 $r\ \beta\ \lambda\ V\ \Gamma\ \Sigma$ - target centered polar coordinates (ecliptic)

The formulas for lines b and c are to be found on page 32. To define the remaining quantities, we use the notations on page 32.

$$C_1 = \sqrt{\mu p} \; ; \qquad C_3 = -\frac{\mu}{a}$$

q, Q, P are defined on page 34 and f on page 32. For the outgoing phase, (e.g., Earth)

$$\bar{V}_\infty = \frac{1}{e \, |a| \, \sqrt{e^2 - 1}} \left[- (\dot{y}_\omega + \sqrt{e^2 - 1} \, \dot{x}_\omega) \, \bar{r} + (y_\omega + x_\omega \sqrt{e^2 - 1}) \, \bar{V} \right]$$

For the incoming phase (e.g., target body)

$$\bar{V}_\infty = \frac{-1}{e \, |a| \, \sqrt{e^2 - 1}} \left[- (\dot{y}_\omega - \sqrt{e^2 - 1} \, \dot{x}_\omega) \, \bar{r} + (y_\omega - x_\omega \sqrt{e^2 - 1}) \, \bar{V} \right]$$

where

$$x_\omega = \frac{p - r}{e} \qquad y_\omega = \sqrt{r^2 - x_\omega^2}$$

$$\dot{x}_\omega = \frac{-y_\omega}{r\sqrt{p}} \qquad \dot{y}_\omega = \frac{x_\omega + er}{r\sqrt{p}}$$

$$\alpha_s = \tan^{-1} \frac{V_{\infty y}}{V_{\infty x}}$$

$$\delta_s = \sin^{-1} \frac{V_{\infty z}}{V_\infty}$$

Geocentric Phase

In the geocentric phase, line 1 is from the orbit, line 2 is given on page 29, line 3 is the same as line 2 except that the velocity components $\dot{x}_e$ and $\dot{y}_e$ replace $\dot{x}$ and $\dot{y}$ on page 29.

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} - \frac{\omega_e a_e}{r} \begin{bmatrix} y \\ -x \end{bmatrix}$$

ω_e = angular rotation rate of earth

a_e = equatorial radius of earth

Heliocentric Phase

Lines 1 to 4 of the heliocentric phase output requires no further explanation. The celestial angles are defined below.

Let r = geocentric position vector of probe

R = heliocentric position vector of probe

p = heliocentric position vector of target planet

e = heliocentric position vector of earth

m = geocentric position vector of moon

s = direction cosines of Star Canopus

All vectors are in the equatorial system unless otherwise indicated. The angles are computed as follows (r' means transpose of r, etc.),

EPS, earth probe sun

$$EPS = \cos^{-1} \frac{r' R}{|r| |R|}$$

EPM, earth probe moon

$$EPM = \cos^{-1} \frac{r' (r-m)}{|r| \, |r-m|}$$

SPC, sun probe Canopus

$$SPC = \cos^{-1} \frac{-R' s}{|R|}$$

TPC, target probe Canopus

$$TPC = \cos^{-1} \frac{s' (p-R)}{|p-R|}$$

SPT, sun probe target

$$SPT = \cos^{-1} \frac{R' \, (R-p)}{|R| \, |R-p|}$$

STP, sun target probe

$$STP = \cos^{-1} \frac{p' \, (p-R)}{|p| \, |p-R|}$$

LOE, Celestial Longitude of Earth

e_y, e_x are in the ecliptic system

$$LOE = \lambda = \tan^{-1} \left(\frac{e_y}{e_x} \right)$$

LOT, Celestial Longitude of Target Planet

p_y, p_x are in the ecliptic system

$$LOT = \tan^{-1} \left(\frac{p_y}{p_x} \right)$$

EMP, earth moon probe

$$EMP = \cos^{-1} \frac{m' \, (m-r)}{|m| \, |m-r|}$$

Target Phase

In the initial print of the target phase

$\overline{B}.\overline{T} = m_1$

$\overline{B}.\overline{R} = m_2$, see page 54 .

The matrix elements M_{ij} are partial derivatives of m_1, m_2, and t_f with respect to the polar velocity coordinates at infinity on the geocentric escape hyperbola. Except for minor modifications, the formulas for these derivatives are found in Appendix 4.

In addition to the foregoing, all the input constants for the trajectory will be printed. The pertinent constants are tabulated in List I of Appendix 9.

APPENDIX 11 INPUT REQUIREMENTS

In order to use the program, a rather large number of input quantities must be specified. The required inputs have been arranged approximately in the sequence in which they are required in performing the functions of the main program blocks, as shown in Figure 1.

In cases where multiple options are available in the program, the option desired must be indicated in the input. Options which control the over-all operation of the program have been arranged into one group termed TAPP Input Controls. Other options which controls specific portions of the program are located in the appropriate input section pertaining to that part of the program.

The TAPP Input Controls are:

1. Input Option Flag
2. Trajectory Class Flag
3. Target Elements Flag
4. Midcourse Maneuver Flag
5. Search Option Flag
6. Observation Type Flags (4)
7. Phase Change Sequence
8. Step Size Control
9. Print Control Option

Let us now consider each control in sequence:

1. Initial Condition Input Option

Initial conditions to the program can be specified by various sets of elements. Each set is identified by a single digit (1, 2, 3, or 7).

2. Trajectory Class Flag

Trajectories are divided into two classes: Class 1 and 2. A Class 1 trajectory is one in which the interception of the target by the spacecraft occurs when the spacecraft first encounters the orbit of the target planet. On a Class 2 trajectory, encounter takes place on the second crossing of the target orbit by the spacecraft. The specification of the class then serves to determine which intercept point is desired.

3. Target Elements Flag

To ensure that end conditions of a mission be as close to reality as possible, one may occasionally desire to input a particular set of osculating elements of the target planet rather than employ the built-in target orbit formulas. A flag at this stage indicates to the program that elements are to be specified as input.

4. Midcourse Maneuver Flag

A flag here will indicate to the program whether a Midcourse Maneuver is or is not to be executed. TAPP, Mod I, provides for the execution only <u>one</u> actual midcourse maneuver in a mission. Any number of "hypothetical" maneuvers as described later can be included. TAPP Mod. II, under development, will allow multiple midcourse and terminal maneuvers to be simulated.

5. Search Option Flag

This flag indicates whether or not a search is to be performed to adjust initial conditions to yield specified terminal conditions. Digit "one" indicates search desired.

6. Observation Type Flags (4)

For any given mission, the spacecraft may or may not be required to make measurements, in addition to the tracking from earth based stations. This "may" or "may not" condition is determined by flags indicating which type of measurements the spacecraft is to make, if any. In addition, a flag is required to indicate whether or not earth based tracking is required.

7. Phase Change Sequence (Up to four phase changes)

The program computes the trajectory of a spacecraft by means of the "patched conic" method. For this reason, it is necessary to specify which planets the mission is likely to encounter, and in which order. As an example, one sequence could be Earth-Moon-Earth-Sun or

Earth-Moon-Sun-Planet. Any sequence up to four is allowable. The following code numbers are used for the central bodies:

1. Earth
2. Sun
3. Moon
4. Venus
5. Mars
6. Jupiter
7. Saturn
8. Arbitrary

The trajectory is not deviated if the vehicle does not enter the planets sphere of action.

8. Step Size Control

This control determines the intervals at which position, velocity and necessary partial derivatives are to be computed along the orbit through control of the increment of eccentric anomaly, in degrees, to be taken. Note that this input does not control the printout interval, but rather the computing step size over which interpolation is used for points called for by the printout interval.

9. Print Control Flag

Controls format and content of printout, are described in Section V of this Appendix, and in Appendix 9.

Following the TAPP Input Controls, the inputs listed below must be supplied:

I. Initial Conditions

The trajectory to be flown may be specified in terms of either injection conditions, or launch conditions:

Injection Conditions

(Option 1) Orbital elements: $t, a, e, i, \Omega, \omega, M_o$

(Option 2) Rectangular coordinates: $t, x, y, z, \dot{x}, \dot{y}, \dot{z}$

(Option 3) Spherical coordinates: $t, r, \alpha, \delta, V, \Gamma, \Sigma$

All angles are given in degrees. The units of length and time can be given independently and are entered by choosing a two digit code from the following list:

Code	Length	Code	Time
1.	Feet	1.	Seconds
2.	Nautical Mile	2.	Minutes
3.	Kilometer	3.	Hours
4.	Earth Radii	4.	Days
5.	Astronomical Unit	5.	Kiloseconds
6.	Megameter		

Launch Conditions (Option 7)

The inputs to this option are:

1. Launch Conditions

   Launch Azimuth (inertial) in degrees
   Launch site Latitude (geodetic) in degrees, positive north
   Launch site Longitude in degrees, positive east

2. V_{∞} and Injection Condition Specifications

   Flight Path Angle (inertial), in degrees
   Vis-Viva Energy, C_3, in $(km/sec)^2$
   Geocentric Radius, in megameters
   Declination of Asymptote, δ_s, in deg., positive north
   Right Ascension of Asymptote, α_s, in deg, positive east

3. Coast and Burn Conditions

   Time from Launch to Parking Orbit Injection in Seconds
   Ti of Second Burn in Seconds
   Angle Swept Out Between Launch and Parking Orbit Injection, in degrees
   Angle Swept Out During Second Burn, in degrees
   Time (Gregorian Date of Launch)

II. Target Conditions and Search Control

A. Target Conditions

If search is to be performed, then target conditions may be specified as:

1. M_1 and M_2 and time of flight where:

Impact parameters M_1 and M_2 are in megameters and, Time of flight in days, or,

2. M_1 and M_2 only

If the orbit is elliptical, then the final M_1 and M_2 are specified by the right ascension and declination with respect to planet.

B. Search Control

The parameters the program varies to meet the specified target conditions of A (See Appendix 9, Section (3); Target and Search Conditions) may either be

1. Initial Conditions

a) the parameters specified may be any (2) or (3) of the 6 initial conditions of position and velocity or,

b) if the parameters are unspecified by the user, then the program will select the parameters with the smallest magnitude of change.

or, 2. V_∞ Vector - The parameters may be

a) Unspecified. The program will change all 3 components if 3 target conditions are specified, or 2 components yielding the least magnitude of change if 2 target conditions are specified, or

b) Specified. The user specifies any 2 of the 3 components of the V_∞ vector.

3. Scale Factors (6), one for each of the initial conditions. These are the weighting factors used when the program is allowed to choose which initial conditions to vary.

III. Observables:

All options here may be used simultaneously, and according to the flags certain or all of the options can be deleted from a computer run.

A. Earth Based Trackers (Type A Observations)

1. Station Number

A two digit number will specify the station longitude, latitude and altitude from a prestored table. New station coordinates (e.g. lunar based stations) may be entered in the table if desired.

2. Station Covariance Model

A one digit number will select the prestored station uncertainty covariance matrix to be used for the given station.

3. Station Limitations

A one digit number selects maximum range and minimum elevation angle limitations from a pre-stored table. A zero signifies "no limitations".

4. Noise Model

A one digit number will specify a certain stored noise model for the station. This noise model specifies the variances associated with observation types.

5. Data Type

A one digit number will signify to the program the observation type the flagged station is to measure. The data types available are:

Code	Type
1	Hour Angle
2	Declination
3	Elevation
4	Azimuth
5	Two-way Doppler
9	Slant Range

6. Least Square Weighting Option (1) or (2)

Option (1) sets all $f_{ij} = 1$ and $\omega_{ij} = 0$ for the final weighting.

Option (2) weights according to input f_{ij} and ω_{ij} (from 7 below).

7. Least Square Weights $(f_{ij}$ and $\omega_{ij})$

The weights f_{ij} and ω_{ij} are used in the program to manufacture the final elements to be used in the weighting matrix. The i th index corresponds to the station number and the j th index corresponds to the data type. (See 1 and 5, respectively, above for description). The numerical values of the weights will be entered as inputs.

8. Starting Day

Starting day, measured from 0^h UT on launch date, will indicate to the program the day the tracking is to begin.

9. Waiting Time

Tracking will begin this number of minutes after the spacecraft has risen over a particular station.

10. End Day

Date on which tracking is to cease, in days from 0^h on launch date.

11. Waiting time before set

Tracking will cease this many minutes before the spaçecraft sets over a particular station.

12. Day Intervals

This input is the spacing between the tracking periods in days. For example, 1 = track visible passes every day; 3 = track visible passes every third day.

13. Sampling Rate

This input in seconds will indicate the rate at which the observations are taken by the tracking station.

Note: A maximum of three tracking patterns is allowed per station. Should this prove too few in a problem, the same station may be given more than one set of station numbers, for 3 patterns/station number.

B. Clock and Cone Angles (Type B Observations)

1. Observation of Star (1) or (2) (Choice of one)

(1) An input in the form of a flag will signify that Canopus, whose right ascension and declination are stored in the computer is to be observed by the vehicle during the mission according to the specified pattern.

(2) An input in the form of a flag here will signify that another star has been selected and the position in the form of right ascension (hours, min, sec) and declination (degrees, minutes, seconds) are to be provided as input.

2. Observation of Planet (Choice of one)

Input here will be an integer signifying which of the 6 planets, whose code digit is listed below, is to be selected. The heliocentric positions of the respective planets are available from the ephemeris portion of the program.

(1) Earth
(2) Sun
(3) Moon
(4) Venus
(5) Mars
(6) Jupiter

3. Noise Model Specification

The noise model is to be specified by input and is in the form of:

(1) Variances in the Spacecraft - Planet line in degrees
(2) Variance in the Read-out, also in degrees, and
(3) Proportionality Scale factor for the variance proportional to the planet's angular diameter.

4. Least Square Weighting Option (1) or (2)

This option is the same as the corresponding option for Earth-Based Trackers (Type A Observation).

5. Least Square Weights (f_i and ω_i)

The weights read into the program here are in degrees. f_1 and ω_1 refer to cone angles. f_2 and ω_2 refer to clock angles and the numerical weights follow these designations.

6. Start Time

This input in days and decimal fraction of day measured from 0^{hr} launch signifies to the program the time to begin the measurements.

7. End Time

This time (in days and fractions of day measured from 0^{hr} launch) signifies to the program the instant to cease measurements.

8. Day Interval

This input (in days and fractions of days) plays the same role as the corresponding input for the Earth Based trackers.

C. Planetary Diameters (Type C Observations)

1. Observation of Planet (Choice of one)

Input here will be an integer signifying which of the 6 planets is to be selected. The planet code numbers are as given above for clock and cone angles.

2. Diameter Limitation, λ

This input in units of planetary radii places a lower limitation on the size of the planet's disc as seen from the spacecraft.

3. Noise Model Specification

Size of the variance due to errors in the planetary diameter measurement and constant bias are computed from the following inputs:

(1) A dimensionless proportionality factor, k, in the planetary diameter added to:

(2) A constant bias in degrees.

4. Least Square Weighting Option (1) or (2)

This option is the same as the corresponding option for Earth Based Trackers (Type A Observations).

5. Least Square Weights (f and ω)

This option performs the same function as the corresponding option for Earth Based Trackers (Type A Observations).

6. Measurement Option (1) or (2)

A flag here will either

(1) Begin measurements on time, or
(2) Begin measurements on distance from planet

7. Times at which measurements are to be taken

Up to 30 inputs in days from launch will specify the times at which measurements are to be taken.

D. Occultations (Type D Observations)

"Occultation" as used here means the measurement of the position of a planet's center with respect to a star background. The inputs are:

1. Observation of a Planet (Choice of one) - Digit codes as before.
2. Noise Model Specification
3. Least Square Weighting Option (1) and (2)
4. Least Square Weights
5. Starting Day
6. End Day
7. Day Intervals.

All inputs here have the same function as the corresponding inputs for the Clock and Cone Angles (Type B Observations).

IV. Osculating Elements of Target Planet

A. Position and Velocity (Option 1)

Inputs for the target planet are rectangular coordinates $x, y, z, \dot{x}, \dot{y}, \dot{z}$ at time t. The units of length and time available are as specified in the Initial Conditions Input.

B. Orbital Elements (Option 2)

Inputs for the target planets are given as:

a Semi-major axis

e	Eccentricity
i	Inclination
Ω	Ascending Node
ω	Argument of Perigee
M_o	Mean Anomaly at Epoch

All angles are in degrees and are with respect to ecliptic of epoch except for the moon where the orientation angles are with respect to the equator. The units of length available for a, are as specified in the Initial Conditions Input.

V. Print Control Option

To spare the user from large amounts of unnecessary or unwanted information, options are provided to determine what type of information will be printed. The options are chosen by entering one of the following digit codes:

Code	Option
1	Tracking Only
2	Rise-Set Only
3	Rise-Set and Trajectory
4	Coarse Accuracy
5	Fine Accuracy

The contents of these options are discussed in Appendix 9.

VI. Midcourse Maneuvers

A. Hypothetical Maneuvers

To prescribe a series of maneuvers, one must specify

1. Start Time

Time of the first maneuver measured in days (not necessarily an integer) from 0^{hr} of the initial day.

2. Day Interval

Spacing of the maneuvers in days between the first and last day.

3. End Day

Day of the last maneuver measured in days from 0^h of the initial day.

4. Number of Components to be Nulled (2 or 3)

If two components are to be nulled, the maneuver vector lies in the critical plane.

5. Accuracy Modal Constants - Constants associated with

(1) Shut off error σ_s
(2) Pointing error σ_p
(3) Resolution error σ_r
(4) Autopilot error σ_a

B. Actual Maneuver

1. Time of Actual Maneuver

Days from 0^h of initial day.

2. Items 4., 5. from Option A.

C. Hypothetical Maneuvers Followed by an Actual

1. Item 1. to 5. from Option A
2. Item 1. from Option B

(Time of actual maneuver must be greater than time of last hypothetical.)

II. <u>A Priori Variances and Covariances of Parameters</u>

A. Parameters to be Adjusted (Up to 25)

1. Uncorrelated parameters and their variances.
2. Covariances of correlated parameters. (See Appendix 9, Page 105.)

B. Parameter Uncertainties to be accounted for but not adjusted

1. Uncorrelated parameters and their variances.
2. Covariances of correlated parameters. (See Appendix 9, Page 105.)

APPENDIX 12 PHYSICAL CONSTANTS

The physical constants which are built into the program are tabulated in this appendix. In addition to the ones below, orbital elements for the solar system are to be found in Appendix 2.

unit of length - 10^6 meters (Mm)

unit of time - 10^3 seconds (ks)

mass constant of the earth - μ_e

$$\mu_e = 398.6032 \quad (\mathrm{Mm}^3/\mathrm{ks}^2)$$

mass ratio of sun to earth - μ_s/μ_e

$$\mu_s/\mu_e = 332951.3$$

mass of sun/mass of Venus - μ_s/μ_v

$$\mu_s/\mu_v = 408645$$

mass of sun/mass of Mars - μ_s/μ_M

$$\mu_s/\mu_M = 3088000$$

mass of sun/mass of Jupiter - μ_s/μ_J

$$\mu_s/\mu_J = 1047.39$$

mass of earth/mass of moon - μ_e/μ_m

$$\mu_e/\mu_m = 81.335$$

equatorial radius of earth - a

$a = 6.378165$ Mm

earth's flattening - f

$f = 298.3$

earth's rotation rate - ω_e

$\omega_e = 4.1780742$ (deg/ks)

velocity of light - c_o

$c_o = 2.997925 \times 10^5$ (Mm/ks)

astronomical unit - au

$au = 1.495990 \times 10^5$ Mm

conversion from meters to feet

1 ft = .3048 meter

APPENDIX 13 FLOW DIAGRAMS

This appendix contains several flow diagrams which may be of assistance to users of the TAPP program. A separate document (programmer's manual) containing more detailed flow diagrams, subroutine descriptions, storage locations, etc. will be issued on completion of the coding and checkout of TAPP Mod I.

Flow diagrams are included here for:

a) The Search Routine
b) Orbit Computation Block
c) Rise-Set Routine
d) b-Vector Block
e) Data Processor
f) Accuracy Criterion Block
g) Midcourse Maneuver Block

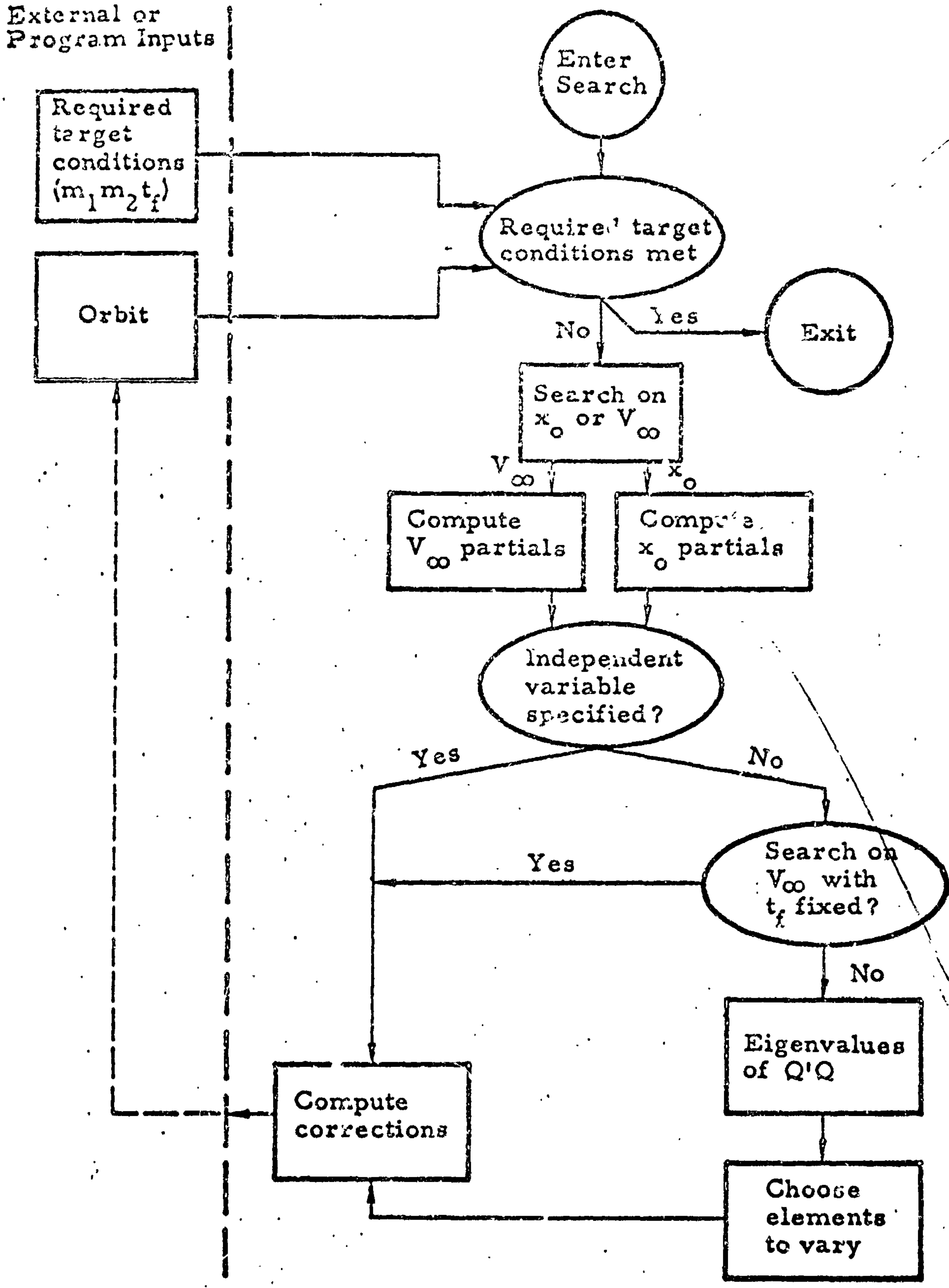

FIGURE 7. Functional Block Diagram of the Search Routine

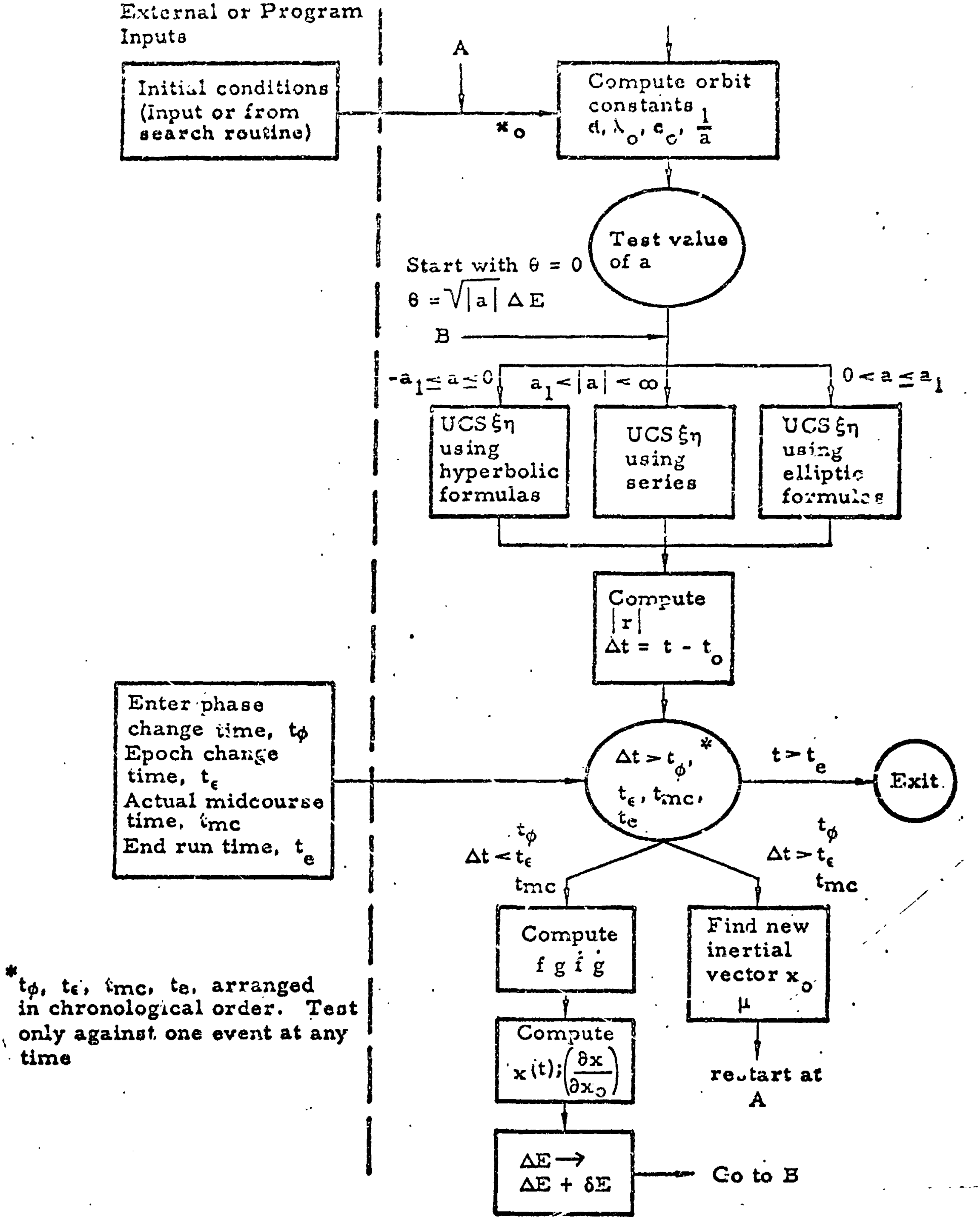

FIGURE 8. Orbit and Variational Equation Block Diagram

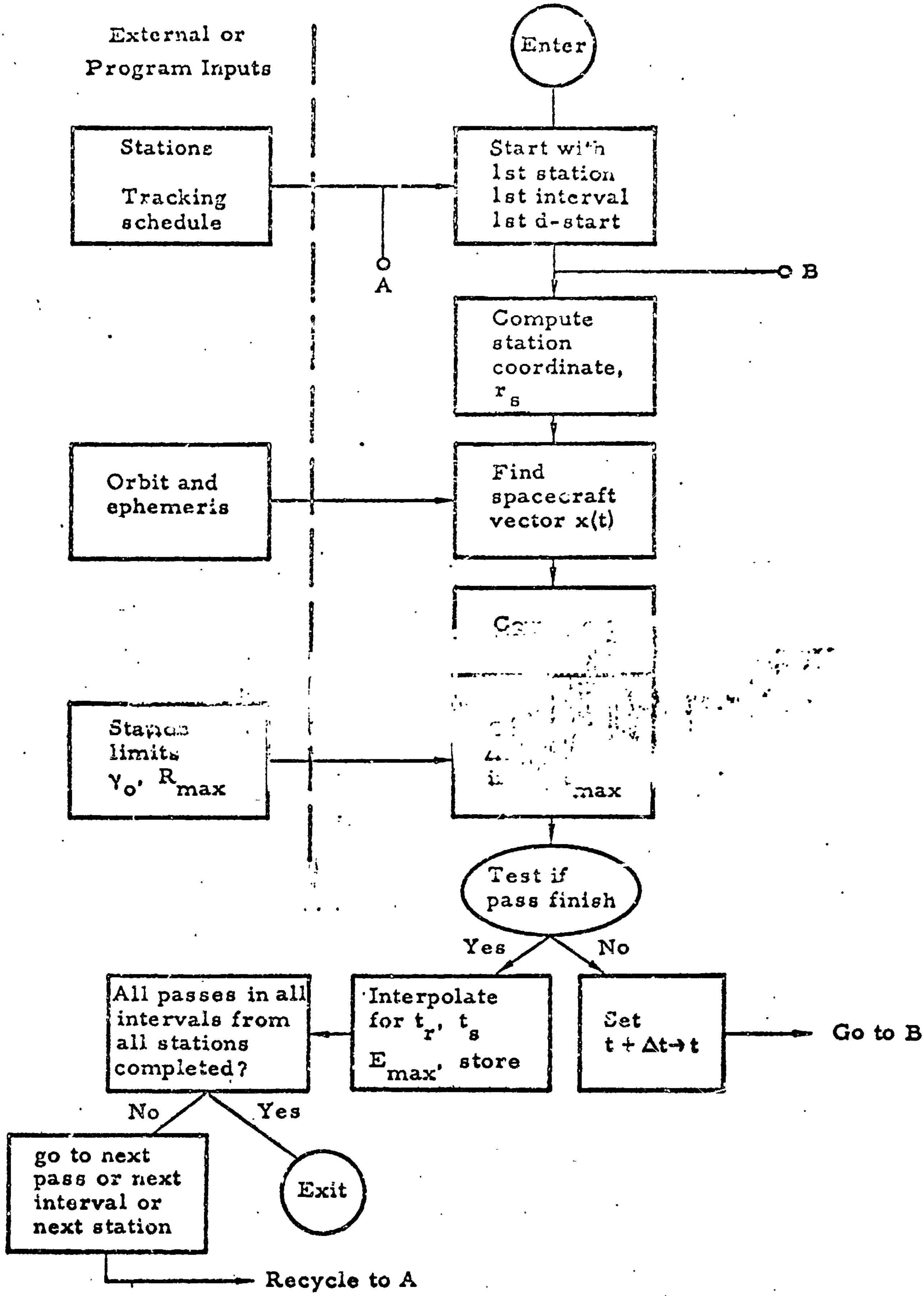

FIGURE 9. Rise-Set Routine Block Diagram

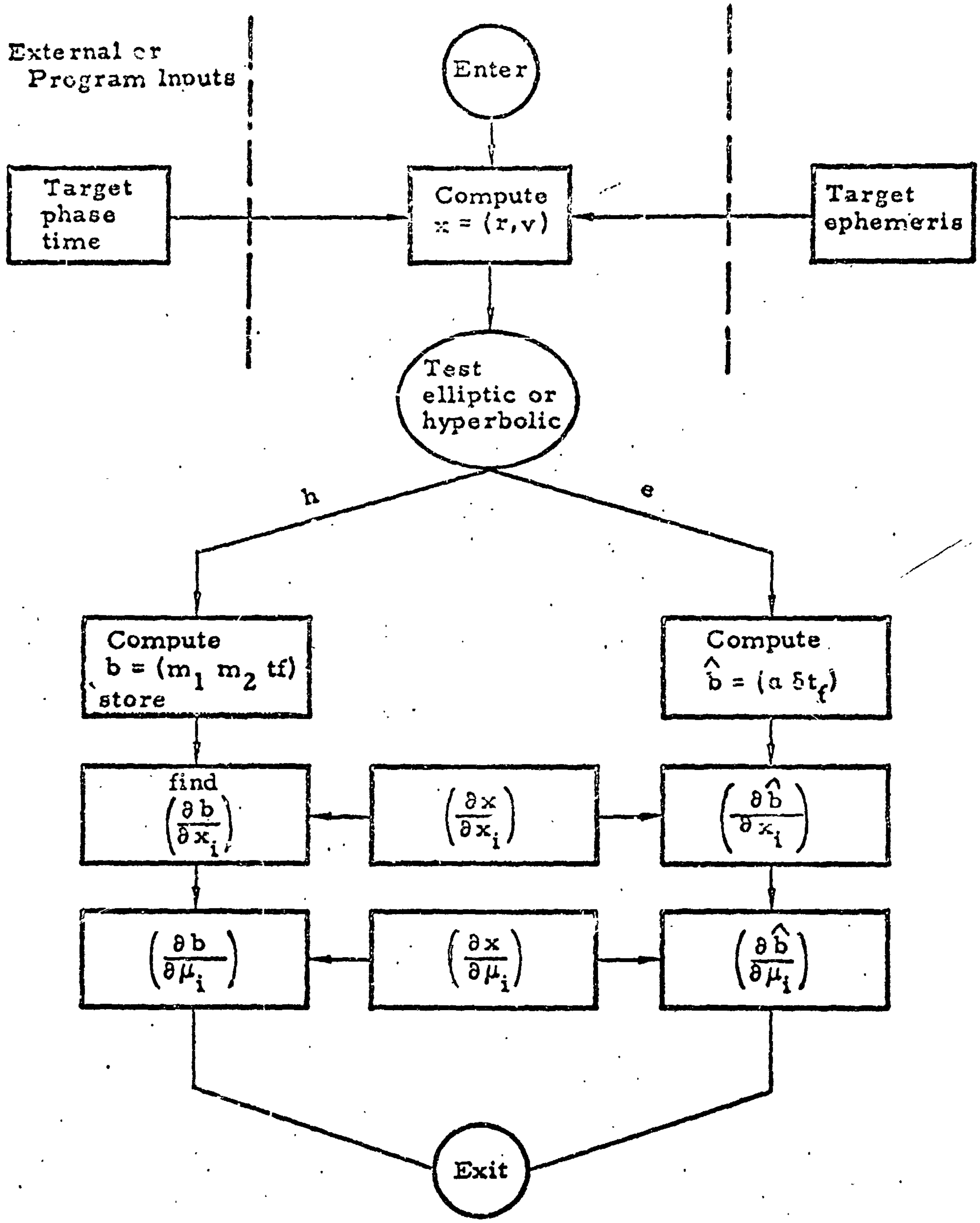

FIGURE 10. b-Vector Block Diagram

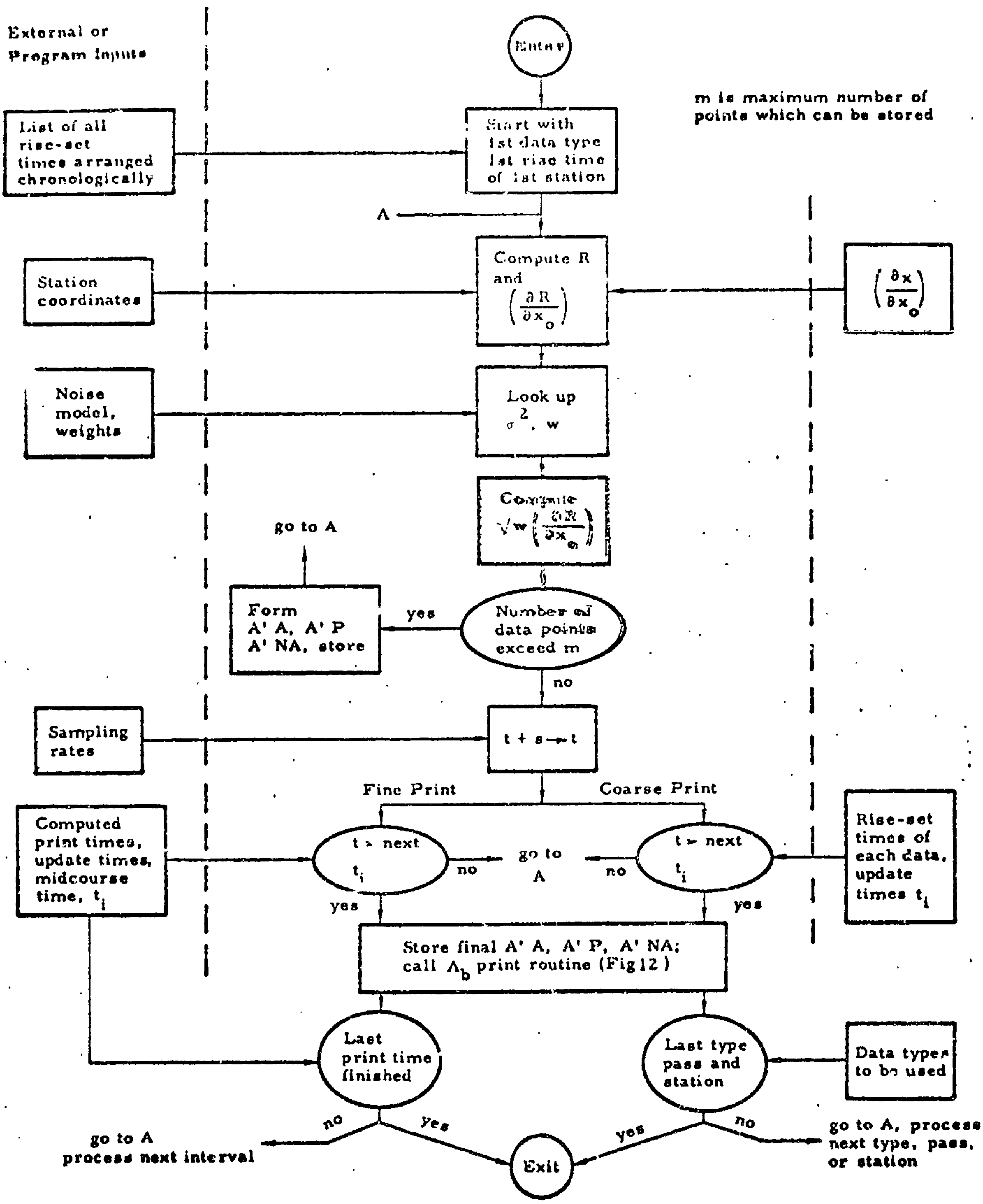

FIGURE 11. Data Processor Block Diagram

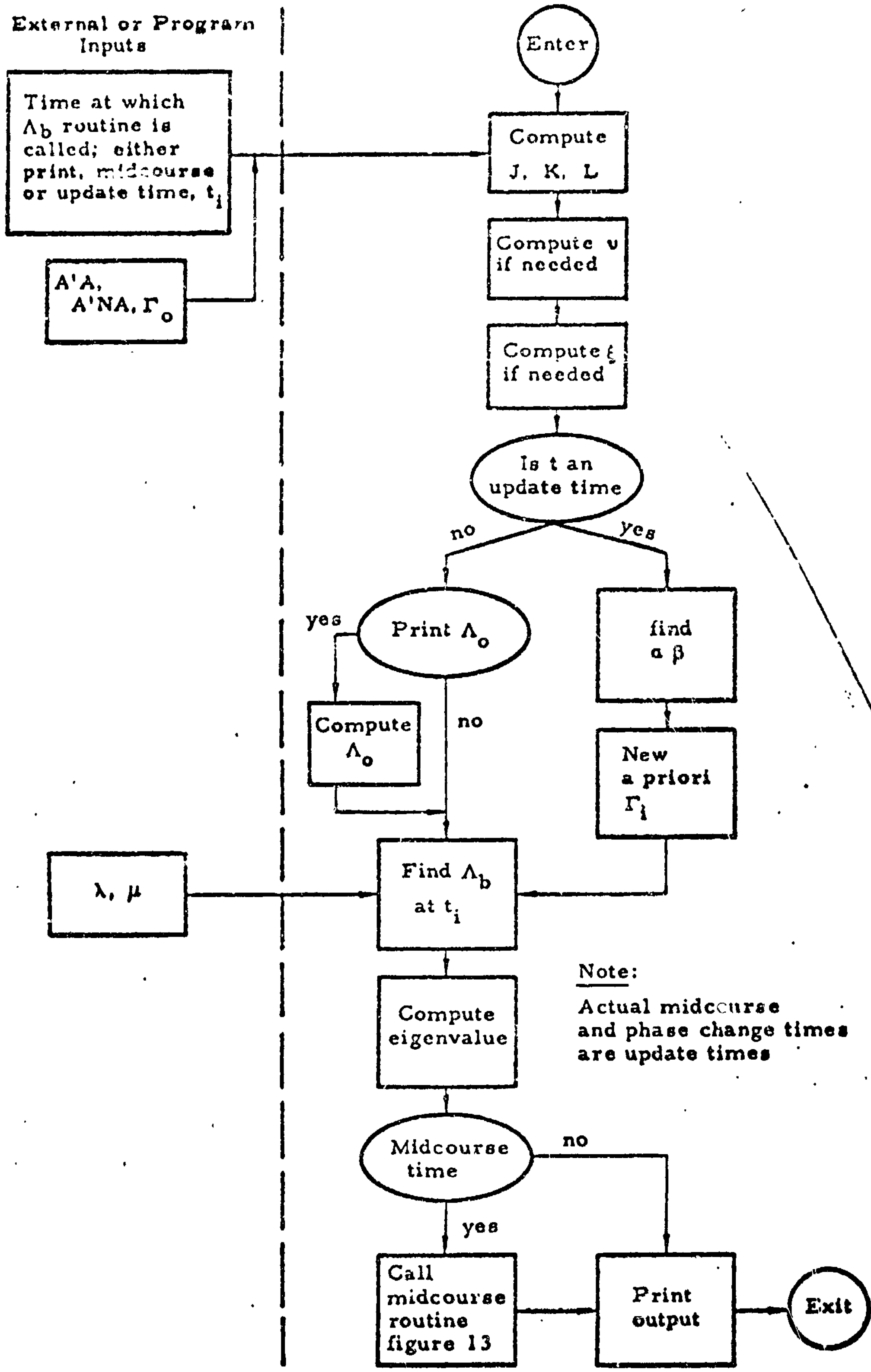

FIGURE 12. Accuracy Criterion Block Diagram

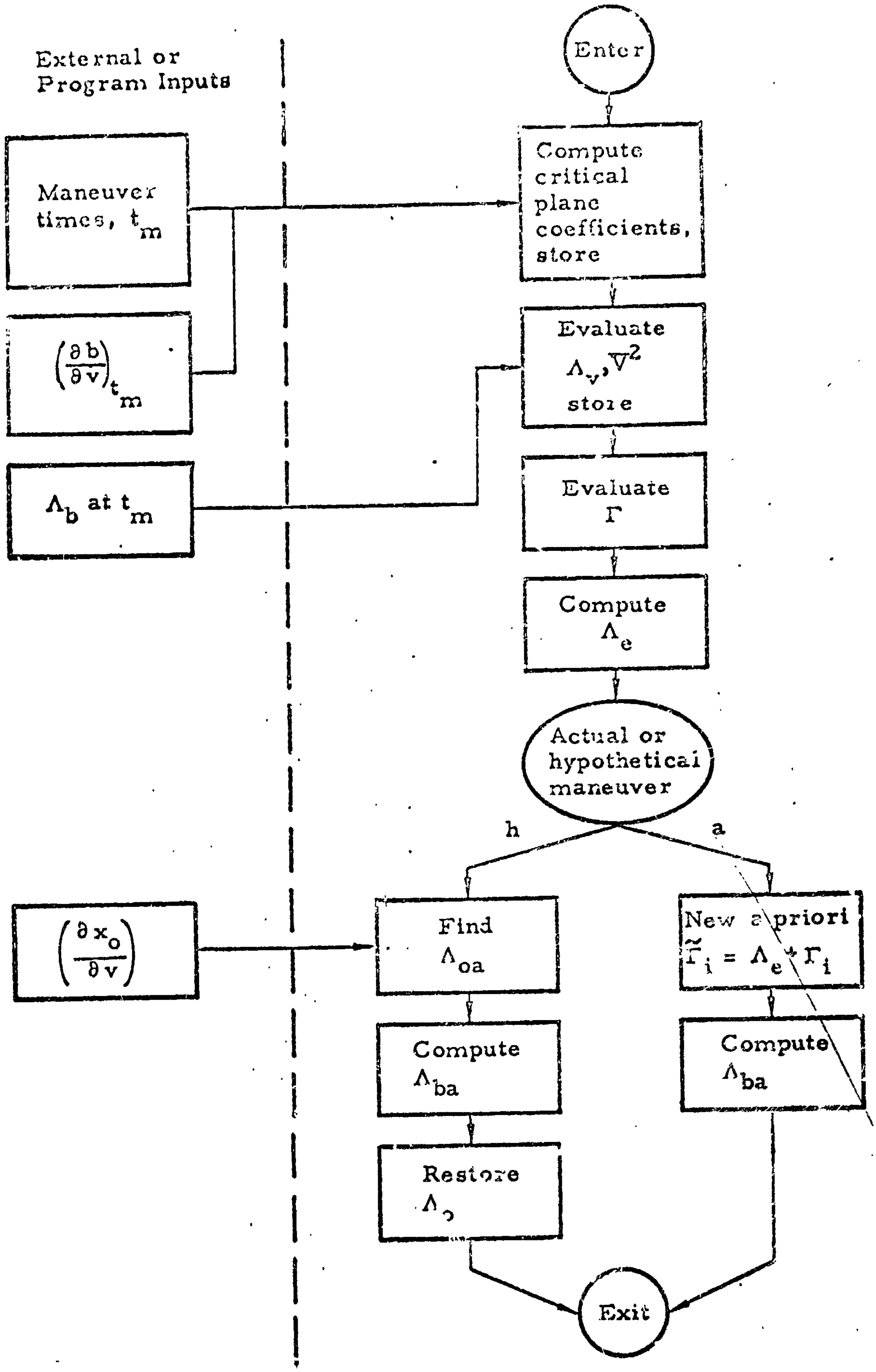

FIGURE 13. Midcourse Maneuver Routine

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